Honors Analysis
Chapter 1
2003

Analysis is the bridge between algebra and trigonometry to calculus. To be effective, we all need to have the same definitions for the same words, the same methods and procedures for the same situations, and the willingness to try *something* even when the solution is not obvious.

# 1 R/C Pg 1 - 11 DO # 2 Q & A to oral ex Pg 4-5 and Pg 9-10 --- # 3 Section I "CakeWalk Algebra" (on back of HW sked) **+ / - intervals on number line Short lecture in NB** 1 / 27	# 4 Pg 5 13 - 18 Pg 10 12 - 19 Pg 10 20 - 25 State Domain ONLY **and** CakeWalk II **Sq roots need the inside to be ≥ 0** 1/28	# 5 Pg 11 26 - 40 replace text instructions with: "Substitute and Simplify" --- #6 R/C Pg 12 - 15 INCLUDE Q & A to Oral ex Pg 16 1 / 29	# 7 Pg 16 - 17 1 - 6, 9, 17 - 26 "normal" = perpendicular For 17-20 you are seeking the point (x,y). We don't do "guess and check" since we are not mathematically challenged! Do talk about analytically 1/30	# 8 "Explain **completely** *EVERYTHING there is to know*" about the 5 functions on Pg 20 and 21. * **Also** # 9 Write answers to Oral exercises Pg 23 pblms 18 through 25 and 32 - 37 **Sgn and Great Int fctns are the new ones** 1 / 31
# 10 Pg 24 - 25 23 - 32 **Create piece-meal fctns for 27 – 32** **Intro to Piece-meal fctns discuss "everything there is to know Show the 1st eg of the demo pblms only**	# 11 CakeWalk III **Show the last 2 demo pblms Give Quiz #1 Have all of previous nights Hw put on boards. Insist on piece-meal versions where possible.** 2 / 4	# 12 Pgs 37 - 38 1 - 20 find G(F(x)) only 21 - 26 find domain-ONLY and CakeWalk IV **First give lecture on Domain of arithmetic and composition then have hw #11 put on boards** 2 / 5	# 13 Pg 45 - 46 1 - 23 odd ; 25 - 27 Pblm 27 is very important! **Range of functions. Inverse Fctns. look closely at yesterday's hw. pblms 25, 26 MUST be factored, #lines or best score is a check. Quiz # 2** 2 / 6	# 14 Pg 45 - 46 2 - 24 even; 28 - 30 Pblm 30 is very important! --- # 15 Pg 49 - 50 4, 5, 6, 8 , 9 , 10 On Pblm 6, **also** make "piece-meal" fctn **Quiz # 3 on yellow paper** 2 / 7
School Holiday $\left(f \circ f^{-1}\right)(x)$ means $f\left(f^{-1}(x)\right)$ 2 / 10	***MAA Exam!!!!*** # 16 ,17 **"Obviously ..."** * **Collect #7 before showing pblm 19; 0 for no effort and check for effort but not solution. Have pg 11 26 - 40 put on boards Slope & Distance**	**Quiz # 4** 2 / 12	Put last year's exam on the board. Come to class prepared! 2 / 13	**EXAM CHAPTER 1** 2 / 14

Honors Analysis
Chapter 1
2003

Analysis is the bridge between algebra and trigonometry to calculus. To be effective, we all need to have the same definitions for the same words, the same methods and procedures for the same situations, and the willingness to try *something* even when the solution is not obvious.

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Honors Analysis

CakeWalk Algebra

Draw number lines and indicate positive or negative as appropriate. Also specifically state positive intervals.

A) $(x+1)$ B) $(x+1)(x-2)$ C) $(x+1)^2(x-2)$ D) $\dfrac{(x-1)(x+3)}{(x-3)}$

E) $\dfrac{(x-1)^2(x+2)}{(x-3)}$ F) $\dfrac{(x-1)(x+2)}{(x+3)(x-3)}$ G) $\dfrac{(x-1)(x+2)}{(x+3)(x-3)^2}$ H) $\dfrac{(x-1)(x+2)}{(x-3)(x-2)^2}$

I) $(x-1)^2$ J) $\dfrac{(x+4)(x-1)}{(x-3)(x-4)}$ K) x^3+1 L) x^4-16

II State the DOMAIN *only*. Graph domain on a number line.

A) $A(x)=\sqrt{\dfrac{x^2-5x+6}{x+1}}$ B) $B(x)=\dfrac{\sqrt{x^2-5x+6}}{x+1}$ C) $C(x)=\dfrac{\sqrt{x-2}}{x^2+x}$

D) $D(x)=\sqrt{\dfrac{x+2}{x-1}}$ E) $E(x)=\sqrt{-x}$ F) $F(x)=\sqrt{9-x^2}$

G) $G(x)=\dfrac{x\sqrt{x}}{x^2-5x+6}$ H) $H(x)=\sqrt{\dfrac{x+3}{x^2+x}}$ I) $H(x)=\dfrac{\sqrt{x+3}}{x^2+x}$

Write each of the following as a "piece-meal function". Graph the function. Specifically state the range.

A) $A(x)=2|x-1|+1$ B) $B(x)=|x+1|\,\mathrm{Sgn}(x-1)$ C) $C(x)=x+x\,\mathrm{Sgn}(x-2)$

D) $D(x)=[\mathrm{Sgn}(x-1)]^2+\mathrm{Sgn}(x+2)$ E) $E(x)=\mathrm{Sgn}(x-2)+|x+2|$ F) $F(x)=|x+|x||$

G) $G(x)=[\mathrm{Sgn}(x+2)][\mathrm{Sgn}(x+2)]$ H) $H(x)=\dfrac{|2x|}{x}$

IV State the DOMAIN *ONLY*. Do **NOT** compute the composition!

Given: $P(x)=\dfrac{x-3}{x^2-6x+9}$ and $Q(x)=\dfrac{x+3}{x^2+3x+2}$

A) $A(x)=\dfrac{P(x)}{Q(x)}$ B) $B(x)=Q(P(x))$ C) $C(x)=P(P(x))$

D) $D(x)=\sqrt{P(x)}$ E) $E(x)=\dfrac{1}{P(x)+Q(x)}$ F) $F(x)=\dfrac{Q(x)}{P(x)}$

G) $G(x)=\sqrt{\dfrac{1}{Q(x)}}$ H) $H(x)=Q(Q(x))$ I) $I(x)=\dfrac{\sqrt{P(x)}}{Q(x)}$

"Obviously..... these are oral exercises.... but we will write the solutions out anyway!"

I Find $f(2+h)$, $f(x+h)$, and $\dfrac{f(x+h)-f(x)}{h}$ (where $h \neq 0$) for

a) $f(x) = x - x^2$ b) $f(x) = \dfrac{x}{x+1}$

II Find the domain of ...

a) $f(x) = \dfrac{x+2}{x^2-1}$ b) $f(x) = \dfrac{x^4}{x^2+x-6}$ c) $f(x) = \sqrt[4]{x^2-6x}$

d) $f(x) = \sqrt[4]{7-3x}$ e) $f(x) = \sqrt[3]{x-1}$

III Find the domain and range and sketch the graph of the function $f(x) = \sqrt{4-x^2}$.

IV Find the domain and sketch the graph of the function:

a) $f(x) = 3-2x$ b) $f(x) = x^2+2x-1$ c) $f(x) = \sqrt{x-5}$

d) $f(x) = \sqrt{6-2x}$ e) $f(x) = x+|x|$ f) $f(x) = |2x|$

g) $f(x) = \dfrac{x}{|x|}$ h) $f(x) = \dfrac{x^2+5x+6}{x+2}$ i) $f(x) = \begin{cases} x & \text{if } x \leq 0 \\ x+1 & \text{if } x > 0 \end{cases}$

j) $f(x) = \begin{cases} 2x+3 & \text{if } x < -1 \\ 3-x & \text{if } x > +1 \end{cases}$ k) $f(x) = \begin{cases} x+2 & \text{if } x \leq -1 \\ x^2 & \text{if } x > -1 \end{cases}$ l) $f(x) = \begin{cases} -1 & \text{if } x < 1 \\ 3x+2 & \text{if } |x| < 1 \\ 7-2x & \text{if } x > 1 \end{cases}$

m) $f(x) = \sqrt{9-x^2}$ n) $f(x) = \sqrt{x^2-5x-6}$ o) $f(x) = \sqrt{x^2-5x+6}$

V Find an expression for the function whose graph is the given curve:

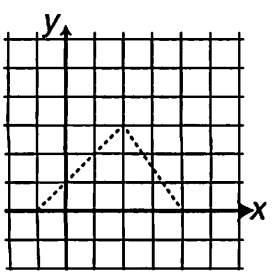
a) The line segment joining the points $(-2, 1)$ and $(4, -6)$.

b) The line segment joining the points $(-3, -2)$ and $(6, 3)$

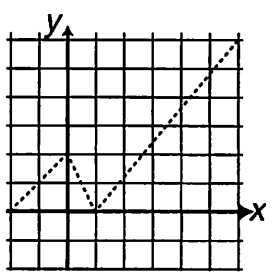
c) The bottom half of the parabola $x + (y-1)^2 = 0$

d) The top half of the circle $(x-1)^2 + y^2 = 1$

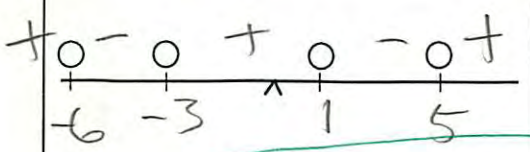
e)



f)

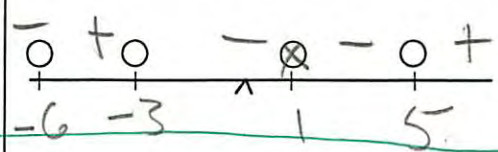


$f(x) = \frac{(x-5)(x+3)}{(x+6)(x-1)}$	$g(x) = \frac{(x-5)(x+3)}{(x+6)(x-1)^2}$	$h(x) = \frac{(x-5)(x+3)^2}{(x+6)^2(x-1)}$	***To Participate or Not To Participate, That is the Question.***
$\frac{-}{-}\frac{-}{-} = \frac{+}{+} = +$	$\frac{-}{-}\frac{-}{+} = \frac{+}{-} = -$	$\frac{-}{+}\frac{+}{-} = \frac{-}{-} = +$	$x < -6$ Pick a Number. Definitely Pick a # in the interval. -100
$\frac{-}{+}\frac{-}{-} = \frac{+}{-} = -$	$\frac{-}{+}\frac{-}{+} = \frac{+}{+} = +$	$\frac{-}{+}\frac{+}{-} = \frac{-}{-} = +$	$-6 < x < -3$ -4
$\frac{-}{+}\frac{+}{-} = \frac{-}{-} = +$	$\frac{-}{+}\frac{+}{+} = \frac{-}{+} = -$	$\frac{-}{+}\frac{+}{-} = \frac{-}{-} = +$	$-3 < x < 1$ 0
$\frac{-}{+}\frac{+}{+} = \frac{-}{+} = -$	$\frac{-}{+}\frac{+}{+} = \frac{-}{+} = -$	$\frac{-}{+}\frac{+}{+} = \frac{-}{+} = -$	$1 < x < 5$ 3
$\frac{+}{+}\frac{+}{+} = \frac{+}{+} = +$	——— = ——— =	$\frac{+}{+}\frac{+}{+} = \frac{+}{+} = +$	$x > 5$ 100



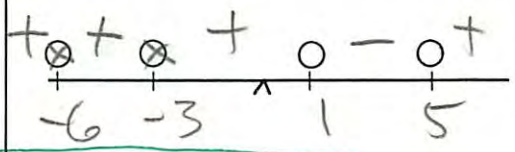
$f(x) \geq 0$

$x < -6$ or $-3 < x < 1$ or $x > 5$



$g(x) \geq 0$

$-6 < x < -3$ OR $x > 5$



$h(x) \geq 0$

Change these to absolute inequalities. Then zero would be included.

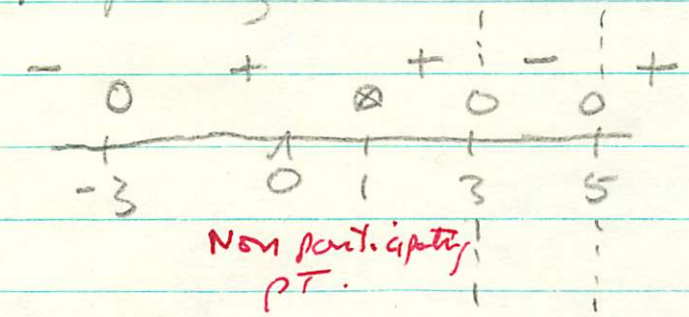
$x < 1, x \neq -6$ or $x > 5$

This is *only* done for the lecture. You will ***not have to write this*** as part of your home work.

1st lecture
analysis
ch 1

$$f(x) = \frac{(x-1)^2(x+3)}{(x-3)(x-5)}$$

① Draw # lines, indicate Positive or NXG intervals
② Specifically state interval



$f(x) > 0 \quad -3 < x < 3,\ x \neq 1,\ x > 5$

$f(x) < 0 \quad x < -3 \quad 3 < x < 5$

must be in factored form!!

HW

ANALYSIS CHAPTER 1 **CAKEWALK ALGEBRA**

I Draw number lines and indicate positive or negative as appropriate. Also, specifically state positive and negative intervals.

A) $(x+1)$ B) $(x+1)(x-2)$ C) $(x+1)^2(x-2)$

D) $\dfrac{(x-1)(x+3)}{(x-3)}$ E) $\dfrac{(x-1)^2(x+2)}{(x-3)}$ F) $\dfrac{(x-1)(x+2)}{(x-3)(x+3)}$

G) $\dfrac{(x-1)(x+2)}{(x-3)^2(x+3)}$ H) $\dfrac{(x-1)(x+2)}{(x-2)^2(x-3)}$ I) $(x-1)^2$

J) $\dfrac{(x+4)(x-1)}{(x-3)(x-4)}$ K) x^3+1 L) x^4-16

II State the **DOMAIN ONLY**. Graph domain on a number line.

A) $A(x) = \sqrt{\dfrac{x^2-5x+6}{x+1}}$ B) $B(x) = \dfrac{\sqrt{x^2-5x+6}}{x+1}$ C) $C(x) = \dfrac{\sqrt{x-2}}{x^2+x}$

D) $D(x) = \sqrt{\dfrac{x+2}{x-1}}$ E) $E(x) = \sqrt{-x}$ F) $F(x) = \sqrt{9-x^2}$

G) $G(x) = \dfrac{x\sqrt{x}}{x^2-5x+6}$ H) $H(x) = \sqrt{\dfrac{x+3}{x^2+x}}$ I) $I(x) = \dfrac{\sqrt{x+3}}{x^2+x}$

III Write the each of the following as a " piece-meal function ". Graph the function. Specifically state the range of the function.

A) $A(x) = 2|x-1|+1$ B) $B(x) = |x+1|\,\mathrm{Sgn}(x-1)$ C) $C(x) = x + x\,\mathrm{Sgn}(x-2)$

D) $D(x) = [\mathrm{Sgn}(x-1)]^2 + \mathrm{Sgn}(x+2)$ E) $E(x) = |x+2| + \mathrm{Sgn}(x-2)$

F) $F(x) = \Big|x + |x|\Big|$ G) $G(x) = [\mathrm{Sgn}(x+2)][\mathrm{Sgn}(x-1)]$ H) $H(x) = \dfrac{|2x|}{x}$

IV State the **DOMAIN ONLY**. Do not compute the composition !

Given : $P(x) = \dfrac{x-3}{x^2-6x+9}$ and $Q(x) = \dfrac{x+3}{x^2+3x+2}$

A) $A(x) = \dfrac{P}{Q}$ B) $B(x) = Q(P)$ C) $C(x) = P(P)$ D) $D(x) = \sqrt{P}$

E) $E(x) = \dfrac{1}{P+Q}$ F) $F(x) = \dfrac{Q}{P}$ G) $G(x) = \sqrt{\dfrac{1}{Q}}$

CW III

A) $f(x) = 2|x-1|+1$

I II III

0 1

I $2(-(x-1))+1$

$2(-x+1)+1$

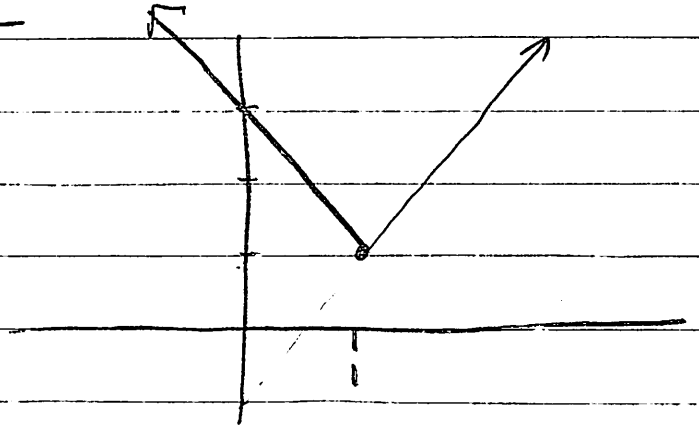
$-2x+3$

II $0+1 = 1$

III $2(x-1)+1$

$2x-2+1$

$2x-1$



$$f(x) = \begin{cases} -2x+3 & x \le 1 \\ 2x-1 & x > 1 \end{cases}$$

B) $f(x) = |x+1| \operatorname{Sgn}(x-1)$

I II III IV V

0 0

-1 0 1

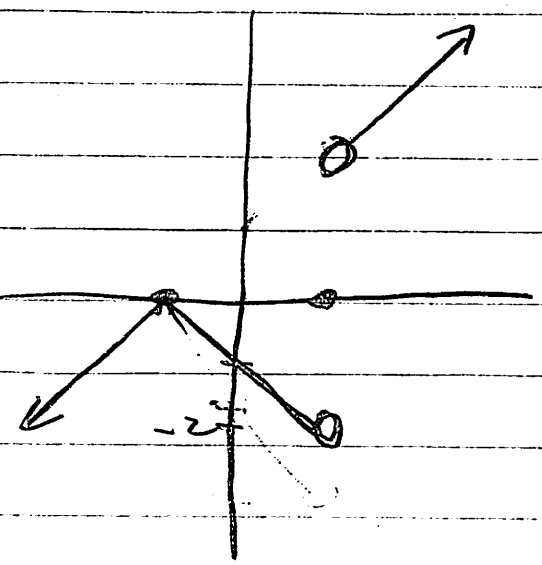
I $-(x+1)(-1) = x+1$

II 0

III $(x+1)(-1) = -x-1$

IV 0

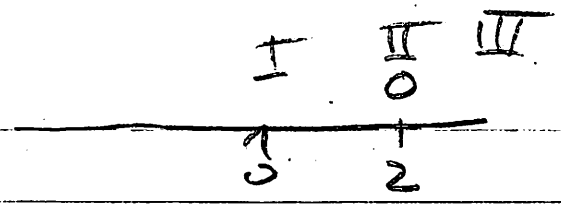
V $x+1$



$$f(x) = \begin{cases} x+1, & x \le -1 \\ -x-1, & -1 < x < 1 \\ 0, & x = 1 \\ x+1, & x > 1 \end{cases}$$

CW III

c) $f(x) = x + x\,\mathrm{sgn}(x-2)$

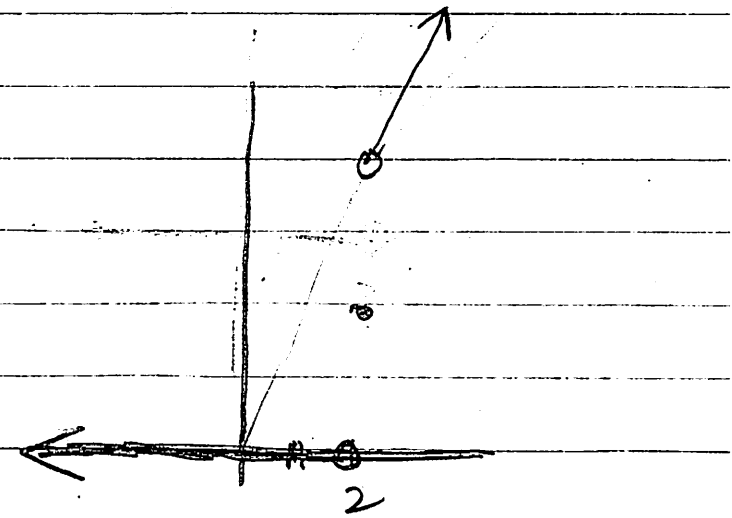


I $x + x(-1) = 0$

II $2 + 0 = 2$

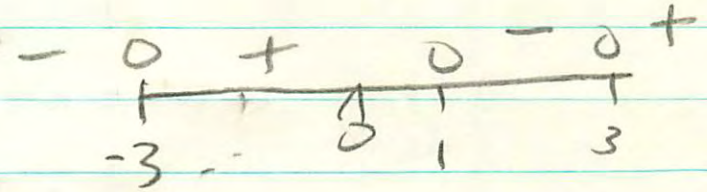
III $x + x = 2x$

$$f(x) = \begin{cases} 0, & x < 2 \\ 2, & x = 2 \\ 2x, & x > 2 \end{cases}$$



[X] Consider $\frac{(x-1)(x+3)}{x-3}$

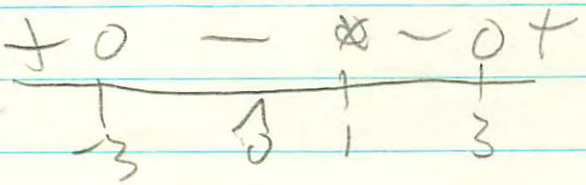
Make # line Specifically state when it is >0.



$-3 < x < 1$ or $x > 3$

Now redem

$$f_B(x) = \sqrt{\frac{(x-1)^2(x+3)}{x-3}}$$



D_B $x \leq -3$ or $x = 1$ or $x > 3$

Ch 1 analysis
2nd Day of class
1ST real lecture.

What is a fctn?

Generic: It is a method of describing a unique result when given a particular item.

algebraic: for a particular Domain item, a fctn describes the process which will lead us to a unique item in the Range.

Discuss and exp 4-5, 9-10

We are most interested in fctns. as associated with the Real numbers.

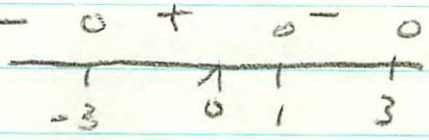
So we must be on guard for two main things

1. Zeros in a denominator
2. Negatives under even radicals.

X

eg Find Domain ONLY!

$$A(x) = \sqrt{\frac{(x-1)(x+3)}{x-3}}$$



D_A $x \geq -3 \leq x \leq 1$ or $x > 3$

Now Do This

$$B(x) = \sqrt{\frac{(x-1)(x+3)}{x-3}}$$

D_B $x \leq -3$ or $x \geq 1$, $x \neq 3$

We only Need ≥ 0 in Radical

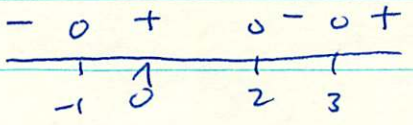
This has nothing to do with radicals

end.

Calkewalk
II

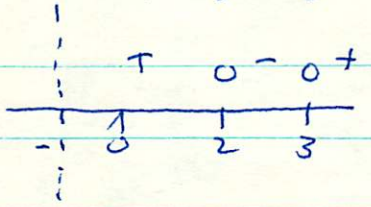
$A(x) = \sqrt{\frac{x^2-5x+6}{x+1}}$

$\frac{(x-3)(x-2)}{x+1}$

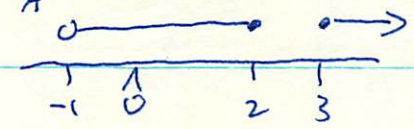


$B(x) = \frac{\sqrt{x^2-5x+6}}{x+1}$

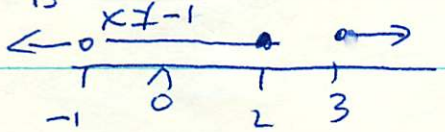
$(x-3)(x-2)$



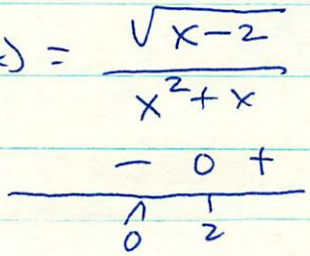
D_A $-1 < x \leq 2$ or $x \geq 3$



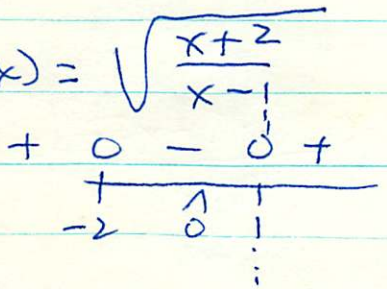
D_B $x \leq 2$ or $x \geq 3$, $x \neq -1$



$C(x) = \frac{\sqrt{x-2}}{x^2+x}$

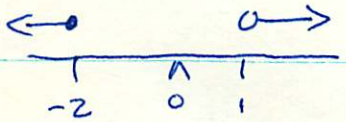


D) $D(x) = \sqrt{\frac{x+2}{x-1}}$



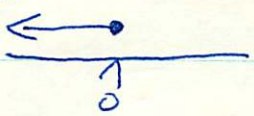
D_C $x \geq 2$

D_D $x \leq -2$ or $x > 1$



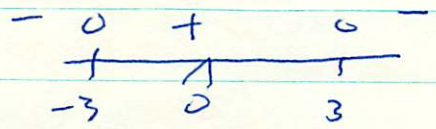
$E(x) = \sqrt{-x}$

D_E $x \leq 0$



$f(x) = \sqrt{9-x^2}$

$(3-x)(3+x)$



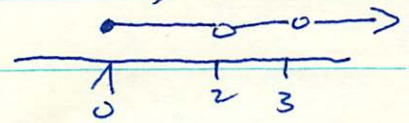
D_F $-3 \leq x \leq 3$ $\{ |x| \leq 3$

Calc walk
II

$$G(x) = \frac{x\sqrt{x}}{x^2-5x+6}$$

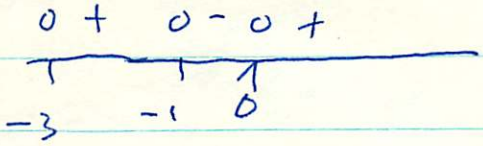
$$(x-2)(x-3)$$

D_G $x \geq 0$, $x \neq \{2, 3\}$



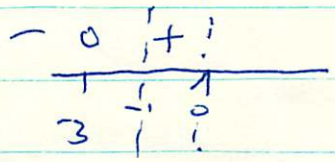
$$h(x) = \sqrt{\frac{x+3}{x^2+x}}$$

$$x(x+1)$$



D_h $-3 \leq x < -1$ or $x > 0$

$$I(x) = \frac{\sqrt{x+3}}{x^2+x}$$



D_f $x \geq -3$, $x \neq \{-1, 0\}$

(13) $\{(0,1),(1,2),(2,3)\}$, $\{(x, x+1)\}$ $x \in (0,1,2)$
Both Describe the same fctn

(14) $\{(x, x+1)\ x \in \{0,1,2\}$ $\{(x-1, x) : x \in \{1,2,3\}\}$ ~~Same~~
Range same Domain Diff :, NOT EXACTLY same.

(15) $\{(x, y) : x \in \mathbb{R}, y = 2x\}$ $\{(2z, z)\ z \in \mathbb{R}$. NO

(16) $\{(x, 2x) : x \in \mathbb{R}\}$ $\{(x^2, 2x^2)\ x \in \mathbb{R}$ NO the 1st fctn contains neg values in range & second doesn't

pg 10

(2) $\{(z, z^2+1)\} ; z \in \mathbb{R}$ Domain $\mathbb{R}$ Range ≥ 1

(6) $I : x \to x$ Range & Domain $\in \mathbb{R}$.

(20) $f(x) = \frac{1}{2}(2x-1)$ $R \& D \in \mathbb{R}$

(21) $\phi(x) = \frac{x}{x+1}$ $D\ x \in \mathbb{R}\ x \neq -1$ $R\ \phi(x) \in \mathbb{R}\ \phi(x) \neq 1$

$x = \frac{z}{z+1}$ $xz + x - z = 0$ $z = \frac{-x}{x-1}$

(22) $h(x) = \frac{x}{x^2+x} = \frac{x}{x(x+1)} = \frac{1}{x+1}$ D_h $x \in \mathbb{R}\ x \notin \{0,-1\}$
R_h $h(x) \in \mathbb{R}$ $h(x) \notin \{0,1\}$

$x = \frac{z}{z^2+z}$ $xz^2 + (x-1)z = 0$ $x = \frac{1}{z+1}$

$z(xz + (x-1)) = 0$ $xz + x = 1$

$z = 0$ or $xz + x - 1 = 0$ $z = \frac{1-x}{x}$
reject this implies $xz = 1 - x$
z is constant $f^{-1}(x) = z = \frac{1-x}{x}$

$\frac{1}{2}$ 3
$\frac{1}{2}(\frac{3}{2})$

$\frac{x}{x^2+x} \overset{\text{set}}{=} 1$ $0 = \frac{1-x}{x}$ $-1 = \frac{1-x}{x}$
$x = 1$ $-x = 1 - x$
$x = x^2 + x$ only when $x = 0$ but $x \neq 0$.

5

it Appears that we need to set $f^{-1} =$ to what we know are its range restrictions in order to find those values which are not in the range of f.

(23) $G(x) = \sqrt{x-1}$ D_g $x \geq 1$ R_g $g(x) \geq 0$

(24) $H(x) = \sqrt[3]{x-1}$ $D, R \in \mathbb{R}$

(25) $K(x) = \sqrt{\dfrac{x}{(x-1)^2}}$ D_K $x \in \mathbb{R}$, $x > 0$ $x \neq 1$

R_K $K(x) \geq 0$

$$x = \sqrt{\frac{z}{(z-1)^2}}$$

$$x^2 = \frac{z}{(z-1)^2}$$

$$x^2(z^2 - 2z + 1) - z = 0$$

$$x^2z^2 - (2x^2+1)z + 1 = 0$$

$$z = \frac{2x^2+1 \pm \sqrt{4x^4+4x^2+1-4x^4}}{2x^2}$$

$$z = \frac{2x^2+1 \pm \sqrt{4x^2+1}}{2x^2}$$

?

$$K(x) = \frac{\sqrt{x}}{x-1} \quad \text{if } x > 1 \qquad K(x) = \frac{\sqrt{x}}{1-x} \quad 0 \leq x < 1$$

$$x = \frac{\sqrt{z}}{z-1}$$

$$(xz - x) = \sqrt{z}$$

F^{-1} Switch, Switch $\dfrac{2x^2+1 \pm \sqrt{4x^2+1}}{2x^2}$

$$0 = 1 \pm \sqrt{4x^2+1}$$

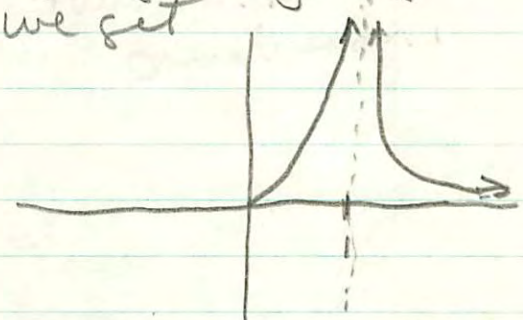
$$-1 = \pm\sqrt{4x^2+1} \quad \text{reject +}$$

$$-1 = -\sqrt{4x^2+1}$$

$$1 = 4x^2+1$$

$$x = 0.$$

After graphing (using calculator) we get



Vertical asymptote at $x = 1$

Fctn is Not 1:1 by horizontal line test.

So Range is ≥ 0

2) $D = \{-2, -1, 0, 1, 2\}$
$R = \{0, 1, 4\}$

3) $D = \{1, 2, 3\}$
$R = \{3\}$

4) $D \Rightarrow x \in \{1, 2, 3, 4, \ldots\}$
$R \Rightarrow y \in \{\ldots, -4, -3, -2, -1\}$

5) $D = \mathbb{R}$
$R = \mathbb{R}$

6) $D = \mathbb{R}$
$R = \mathbb{R}$

7) $D \Rightarrow \mathbb{R}$
$R \Rightarrow y \leq 0$

8) $D = \mathbb{R}$
$R = \mathbb{R}$

9) $D = \{0, 1, 4\}$
$R = \{0, 1, 2\}$

$f(x) \rightarrow x^2 - 1$

12) $f(0) = 0 - 1 = -1$
13) $f(2) = 2^2 - 1 = 3$
14) $f(-2) = 3$
15) $f\left(\frac{3}{2}\right) = \frac{9}{4} - 1 = \frac{5}{4}$

20 $f: x \to \frac{1}{2}(2x-1)$

$x = \mathbb{R}$

21 $\phi: x \to \frac{x}{x+1}$

$x \in \mathbb{R} \quad x \neq -1$

22 $h: x \to \frac{x}{x^2+x}$

$x \in \mathbb{R} \quad x \neq 0, -1$ $\quad$ Rh $\quad x \in \mathbb{R} \quad h \neq 0,$

23 $G: x \to \sqrt{x-1}$

$x \in \mathbb{R} \quad x \geq 1$

24 $H: x \to \sqrt[3]{x-1}$

$x \in \mathbb{R}$

25 $K: x \to \sqrt{\frac{x}{(x-1)^2}}$

$x \in \mathbb{R}, x \neq 1, x \geq 0$

$x^2 = \frac{z}{(z-1)^2}$

$x^2(z^2-2z+1) = z$

$x^2z^2 - 2x^2z + x^2 = z$

$x^2z^2 - (2x^2+1)z + x^2 = 0$

$z = \frac{2x^2+1 \pm \sqrt{4x^4+4x^2+1-4x^4}}{2x^2}$

$z = \frac{2x^2+1 \pm \sqrt{4x^2+1}}{2x^2}$

Discuss the geometric meaning of these pblms. of 36-40

26-31 are just composition of fctns.

(26) $f(a-1)$ $f: x \to (x)(x+1)(x+2)$

$(a-1+1)(a-1+2)$

$(a)(a+1) = a^2+a$ $\quad a \in \mathbb{R}$

27) $f(2c)$ $f: x \to x^2+x+1$

$(2c)^2+2c+1$

$4c^2+2c+1$ $\quad c \in \mathbb{R}$

28) $f(-z)$ $f: x \to x^4+2x^3+3x^2+4x+5$

$z^4-2z^3+3z^2-4z+5$

$z \in \mathbb{R}$

29) $f\left(\frac{1}{y}\right)$ $f: x \to x^2+1$

$\frac{1}{y^2}+1$ $\quad y \neq 0$ $\quad y \in \mathbb{R}$

30) $f\left(\frac{1}{z}\right)$ $f: x \to x+\frac{1}{x}$

$\frac{1}{z}+z$ $\quad z \in \mathbb{R}$ $\quad z \neq 0$

31) $f\left(\frac{1}{z}\right)$ $f: x \to \frac{x}{x+1}$

$$\frac{\frac{1}{z}}{\frac{1}{z}+1} = \frac{\frac{1}{z}}{\frac{1+z}{z}} = \frac{1}{z} \cdot \frac{z}{z+1} = \frac{1}{z+1}$$

$z \in \mathbb{R}$ $z \neq 0, z \neq -1$

if (30) $g = \frac{1}{x}$

$f = x + \frac{1}{x}$

$f(g) = ?$

Domain? etc.

32. $f\left(\frac{1}{a+1}\right)$ $f: x \to \frac{1}{x-1}$

$f: \frac{1}{a+1} \to \frac{1}{\frac{1}{a+1}-1} =$

$\frac{1}{\frac{1-a-1}{a+1}}$

$a \neq 0$
$a \neq -1$

$\frac{1}{\frac{-a}{a+1}}$

$\frac{-(a+1)}{a}$

33 $f\left(\frac{1}{a}+1\right)$ $f: x \to \frac{1}{x-1}$

$\frac{1}{\frac{1}{a}+1-1} = \frac{1}{\frac{1}{a}} = a$

$a \neq 0 \quad a \in \mathbb{R}$

34) $f(x-a)$ $f: x \to x^2 - a^2$

$(x-a)^2 - a^2$

$x^2 - 2ax +$

$a, x \in \mathbb{R}.$

35 $f\left(\frac{x+a}{a}\right)$ $f: x \to \frac{x+1}{x-1}$

$$\frac{\frac{x+a}{a}+1}{\frac{x+a}{a}-1} = \frac{\frac{x+a+a}{a}}{\frac{x+a-a}{a}} = \frac{x+2a}{x}$$

$a \neq 0$, $\frac{x+a}{a} \neq -1$

$x+a \neq a$

$x \neq 0$

$f(2+h)$

36 $f(2-h) - f(2)$ $f: x \to x^2+1$

$1+4+4h+h^2-5$

h^2+4h

$f(2-h) = (2-h)^2+1$ $f(2) = 4+1=5$

$4-2h+h^2+1$

$h^2-2h+5-5 = h^2-2h$,

$h \in \mathbb{R}$.

37, $f(z+h) - f(z)$ $f: x \to x^2+x+2$

$f(z+h) = z^2+2hz+h^2+z+h+2$ $f(z) = z^2+z+2$

$z^2+2hz+h^2+z+h+2-z^2-z-2$

$2hz+h^2+h$

38. $\frac{f(x+h)-f(x)}{h}$ $f(x) = x^2$

$\frac{x^2+2xh+h^2-x^2}{h} = 2x+h$

39 $\frac{f(x+h)-f(x)}{h} = f(x)=x^2-x+2$

$\frac{x^2+2hx+h^2-x-h+2-x^2+x-2}{h}$

$\frac{2hx-h+h^2}{h} = 2x-1+h$

40 $\frac{f(x+h)-f(x)}{h} = \quad f(x)=\frac{1}{x}$

$f(x+h) = \frac{1}{x+h}$

$f(x) = \frac{1}{x}$

$\frac{\frac{1}{x+h}-\frac{1}{x}}{h} = \frac{x-x-h}{x(x+h)h} = \frac{-1}{x(x+h)}$

1-3 Cartesian coordinates

$\mathbb{R} \times \mathbb{R}$ Read R cross R. means all points contained w/in 2 Dimentions graph.

Distance formula $D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Slope formula $\frac{y_2 - y_1}{x_2 - x_1} = m$

two Straight lines do one of the 4 following.

1. intersect $m_1 \neq m_2$
2. Perpendicular $m_1 = \frac{-1}{m_2}$
3. Parallel $m_1 = m_2$ Diff intercepts
4. Same line $m_1 = m_2$ same intercepts

① $P_1(0,0)$ $P_2(3,4)$

$\frac{4-0}{3-0} = \frac{4}{3} = m$ $\qquad \sqrt{(4-0)^2+(3-0)^2} = 5$

③ $P_1 = (-1,-1)$ $P_2(2,3)$

$\frac{3+1}{2+1} = \frac{4}{3} = m$ $\qquad \sqrt{(2+1)^2+(3+1)^2} = 5$

⑤ $P_1 = (2,-3)$ $P_2 = (-1,0)$

$\frac{-3-0}{2+1} = -1 = m$ $\qquad \sqrt{(2+1)^2+(-3+0)^2} = \sqrt{13}$

⑦ $A(5,-3)$
$b(-1,-1)$
$c(3,1)$

$\frac{-3+1}{5+1} = \frac{-2}{6} = -\frac{1}{3}$ $\qquad \frac{-3-1}{5-3} = \frac{-4}{2} = -2$

$\frac{1+1}{3+1} = \frac{2}{4} = \frac{1}{2}$

Rt Δ.

(ab) $\sqrt{(5+1)^2+(-3+1)^2} = \sqrt{36+4} = 2\sqrt{10}$

(bc) $\sqrt{(-1-3)^2+(1+1)^2} = \sqrt{16+4} = 2\sqrt{10}$

(ac) $\sqrt{(5-3)^2+(-3+1)^2} = \sqrt{4+4} = 2\sqrt{2}$

Isosceles rt Δ.

17

13. A(0,2)
B(5,4)
C(2,-1)
D(-3,-3)

$m(ab) = \frac{4-2}{5-0} = \frac{2}{5}$

$m(bc) = \frac{4+1}{5-2} = \frac{5}{3}$

$m(cD) = \frac{-1+3}{2+3} = \frac{2}{5}$

$m(AD) = \frac{2+3}{0+3} = \frac{5}{3}$

$D(ab) = \sqrt{(0-5)^2 + (2-4)^2} = \sqrt{25+4} = \sqrt{29}$

$D(bc) = \sqrt{(5-2)^2 + (4+1)^2} = \sqrt{9+25} = \sqrt{34}$

D(

Parallelogram

17 A(0,0) $m = \frac{3}{4}$ $d = 5$

$m = \frac{y_1 - y_2}{x_1 - x_2}$

$\frac{3}{4} = \frac{y-0}{x-0}$

$d = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$

$x = 4$ $y = 3$ or $x = -4$ $y = -3$

$d = \sqrt{4^2 + 3^2}$

$d = 5$

19 A(1,2) $m = -\frac{4}{3}$ $d = 10$

$\frac{2-y}{1-x} = -\frac{4}{3}$

$6 - 3y = -4 + 4x$

$4x = 10 - 3y$

$x = \frac{10-3y}{4}$

$10 = \sqrt{(1-x)^2 + (2-y)^2}$

$10 = \sqrt{\left(1 - \frac{10-3y}{4}\right)^2 + (4 - 4y + y^2)}$

$10 = \sqrt{\left(\frac{4-10+3y}{4}\right)^2 +$

$= \sqrt{\left(\frac{3y-6}{4}\right)^2 + 4 - 4y + y^2}$

$\sqrt{\frac{9y^2 - 36y + 36}{16} + 4 - 4y + y^2}$

#19 $A(1,2)$ $m=-\frac{4}{3}$ $d=10$

$-\frac{4}{3}=\frac{y-2}{x-1}$

$4-4x=3y-6$

$10-4x=3y$

$\frac{10-4x}{3}=y$

$10=\sqrt{(x-1)^2+(y-2)^2}$

$10=\sqrt{x^2-2x+1+\left(\frac{10-4x-6}{3}\right)^2}$

$100=x^2-2x+1+\left(\frac{4-4x}{3}\right)^2$

$900=9x^2-18x+9+16-32x+16x^2$

$900=25x^2-50x+25$

$25x^2-50x-875=0$

$x^2-2x-35=0$

$(x+5)(x-7)$

$x=-5$ $\quad$ $x=7$

$y=10$ $\quad$ $y=-6$

$25\overline{)875}$ $\quad$ 35, 75, 125

$\frac{10+20}{3}=y=10$

$\frac{10-28}{3}=-6$

Better method.

$-\frac{4}{3}=\frac{y-2}{x-1}$

$-\frac{4}{3}(x-1)=y-2$

$10=\sqrt{(x-1)^2+(y-2)^2}$

$10=\sqrt{(x-1)^2+\frac{16}{9}(x-1)^2}$

$=\sqrt{\frac{25}{9}(x-1)^2}$

$=\pm\frac{5}{3}(x-1)$

$\pm 10=\frac{5}{3}(x-1$

$\pm 6=x-1$

$x=1\pm 6$

$x\in\{-5,7\}$

17

20 $A(2,2)$ $m=2$ $d=2$

$2 = \frac{y-2}{x-2}$ $\qquad 2 = \sqrt{(y-2)^2 + (x-2)^2}$

$2x-4 = y-2$

$2x-2 = y$

$2 = \sqrt{(2x-2-2)^2 + x^2-4x+4}$

$4 = (2x-4)^2 + x^2-4x+4$

$4 = 4x^2-16x+16+x^2-4x+4$

$5x^2-20x+16 = 0$

$\frac{20 \pm \sqrt{400-320}}{10}$ $\qquad 16 \cdot 5$

$x = \frac{20 \pm \sqrt{80}}{10}$

$x = \frac{20 \pm 4\sqrt{5}}{10} = \frac{10 \pm 2\sqrt{5}}{5}$

$2\left(\frac{10+2\sqrt{5}}{5}\right) - 2 = y$

$\frac{20+4\sqrt{5}-10}{5} = y$

$\frac{10+4\sqrt{5}}{5} = y$ $\quad$ or $\quad \frac{10-4\sqrt{5}}{5}$

$(-3,0)\quad(1,2)$

$$m = \frac{2-0}{1+3} = \frac{2}{4} = \frac{1}{2}$$

21 $(0,0), (1,T)$

$$\frac{T-0}{1-0} = \frac{1}{2}$$

$$T = \frac{1}{2}$$

Parallel.

$T = -2$

Perpendicular.

22 $(0,2)(2,T)$

$$\frac{T-2}{2-0} = \frac{1}{2}$$

$$\frac{T-2}{2} = \frac{1}{2}$$

$$T-2 = 1$$

$$T = 3$$

$$\frac{T-2}{2} = -2$$

$$T-2 = -4$$

$$T = -2$$

23 $(3,-2), (4,T)$

$$\frac{T+2}{4-3} = \frac{1}{2}$$

$$T+2 = \frac{1}{2}$$

$$T = -\frac{3}{2}$$

$$\frac{T+2}{4-3} = -2$$

$$T+2 = -2$$

$$T = -4$$

$$\frac{-2-T}{1} = -\frac{1}{2}$$

$$-2-T = -\frac{1}{2}$$

$$-T = \frac{3}{2}$$

$$T = \frac{3}{2}$$

24 $(0,2)\ (\tau,\tau)$

$\frac{\tau-2}{\tau} = \frac{1}{2}$ $\qquad$ $\frac{\tau-2}{\tau} = -2$

$\tau-2 = \frac{\tau}{2}$ $\qquad$ $\tau-2 = -2\tau$

$2\tau-4 = \tau$ $\qquad$ $3\tau = 2$

$\tau = 4$ $\qquad$ $\tau = \frac{2}{3}$

25 $(1,1),\ (\tau+1, \tau-1)$

$\frac{\tau-1-1}{\tau-1+1} = \frac{1}{2}$ $\qquad$ $\frac{\tau-2}{\tau} = -2$

$\tau = \frac{2}{3}$

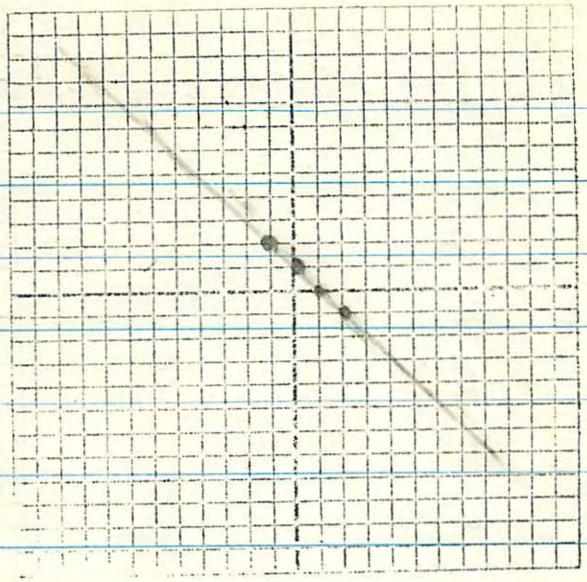
$\frac{\tau-2}{\tau}$ Same as 24

26 $(\tau,\tau)(\tau+1, 2\tau)$

$\frac{2\tau-\tau}{\tau+1-\tau} = \frac{1}{2}$ $\qquad$ $\tau = -2$

$\frac{\tau}{1} = \frac{1}{2}$

① $\{(-1,2),(0,1),(1,0),(2,-1)\}$



⑤ $\{(x, 4-x^2) : x \in \{-2,-1,0,1,2\}\}$

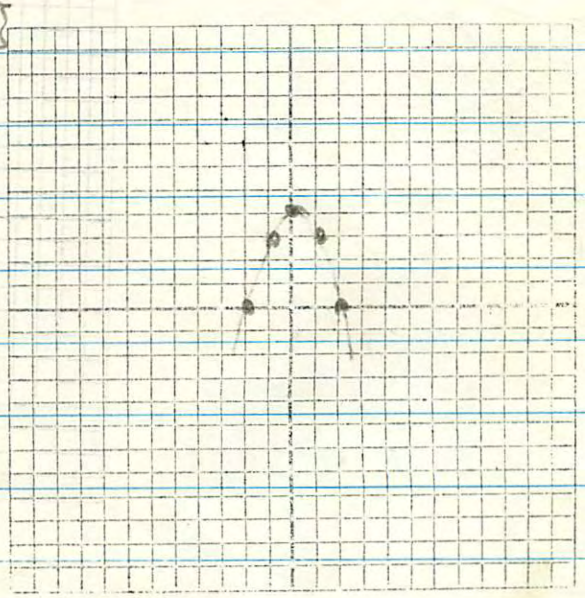
$-2, 0$

$-1, 3$

$0, 4$

$1, 3$

$2, 0$



(6) $\{x, [x]: x \in \{-1\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, 1\frac{1}{2}, 2\frac{1}{2}\}$

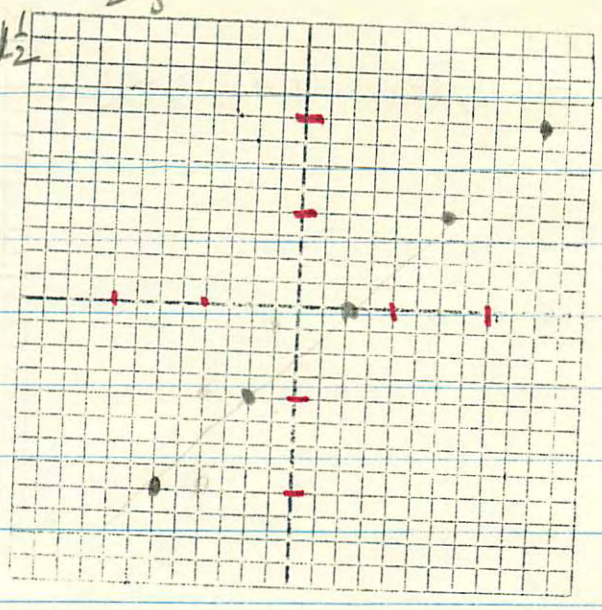
$-1\frac{1}{2}$ -2

$-\frac{1}{2}$ -1

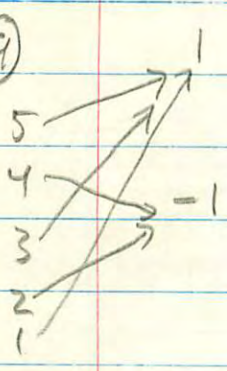
$\frac{1}{2}$ 0

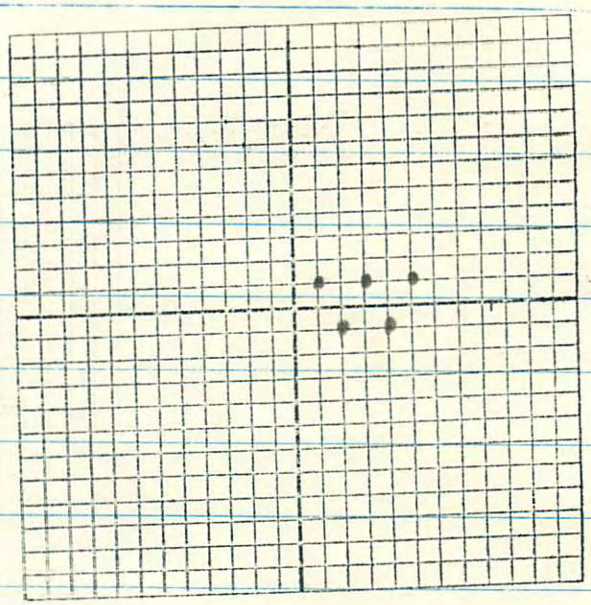
$1\frac{1}{2}$ 1

$2\frac{1}{2}$ 2



(9)

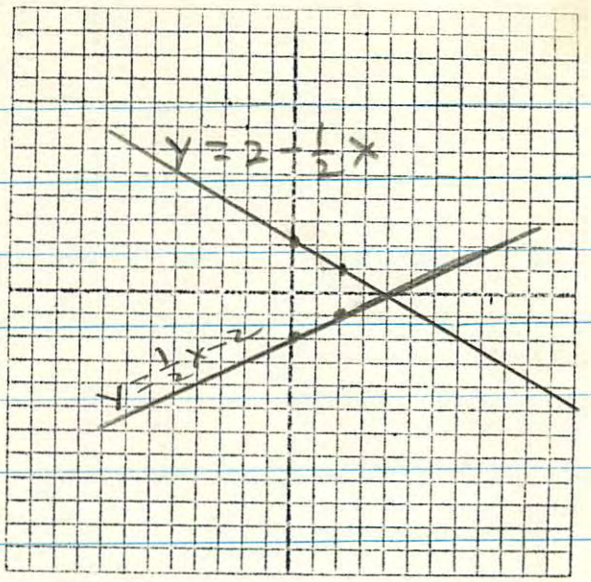




11. $x+2y-4=0$

$2y=4-x$

$y=2-\frac{1}{2}x$



12. $x-2y-4=0$

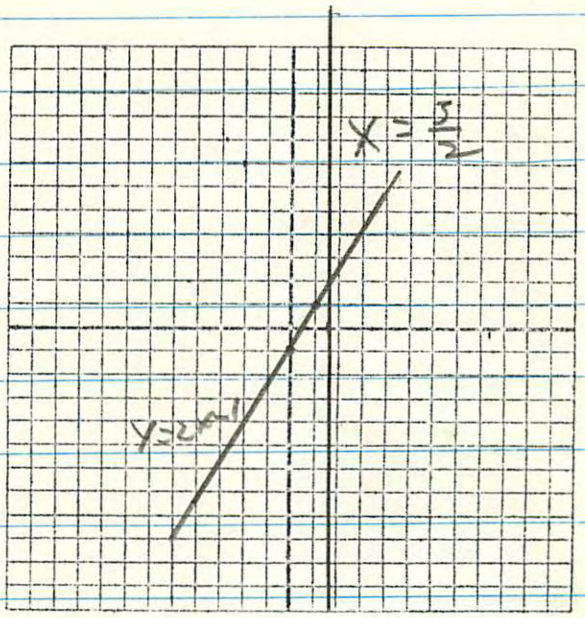
$2y=x-4$

$y=\frac{1}{2}x-2$

13 $2x-3=0$

$2x=3$

$x=\frac{3}{2}$



14 $y=2x-1$

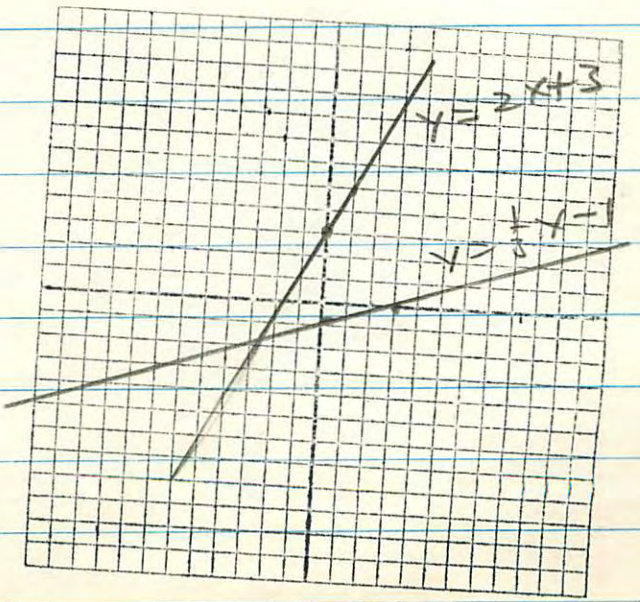
#15 $y-1=2(x+1)$

$y-1=2x+2$

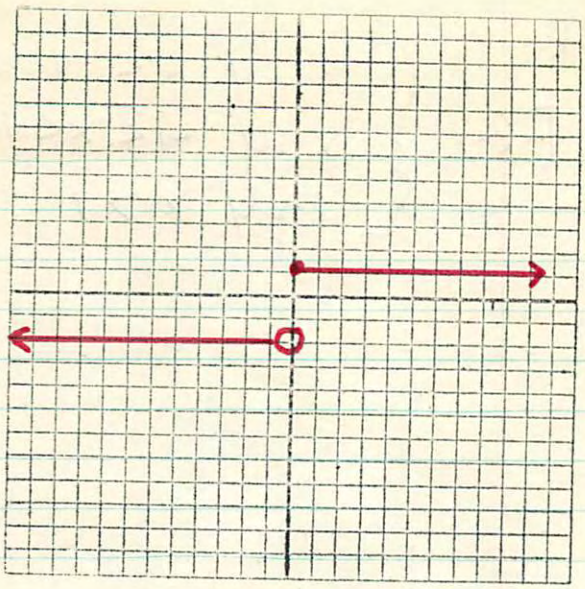
$x=2x+3$

#16 $\frac{x}{3}-y=1$

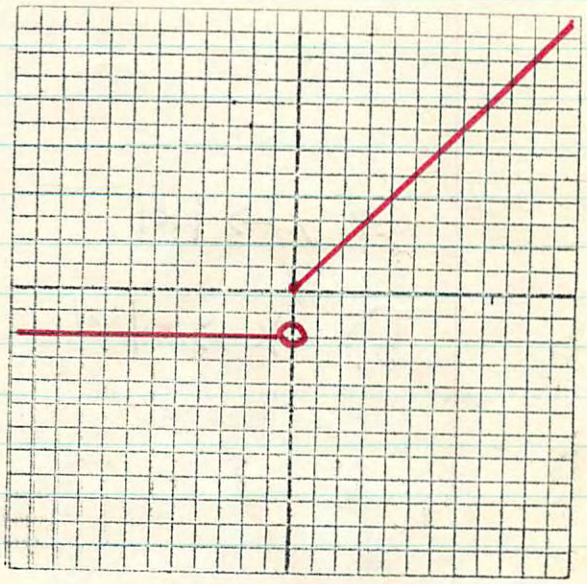
$y=\frac{1}{3}x-1$



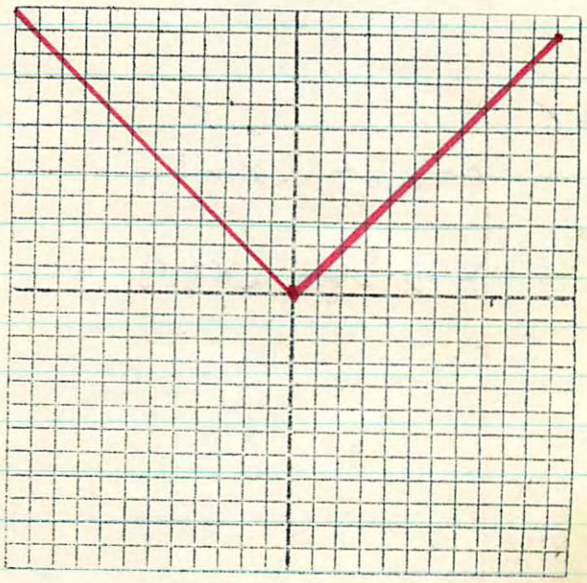
23 $f(x) \rightarrow \begin{cases} 1 \text{ if } x \geq 0 \\ -2 \text{ if } x < 0 \end{cases}$

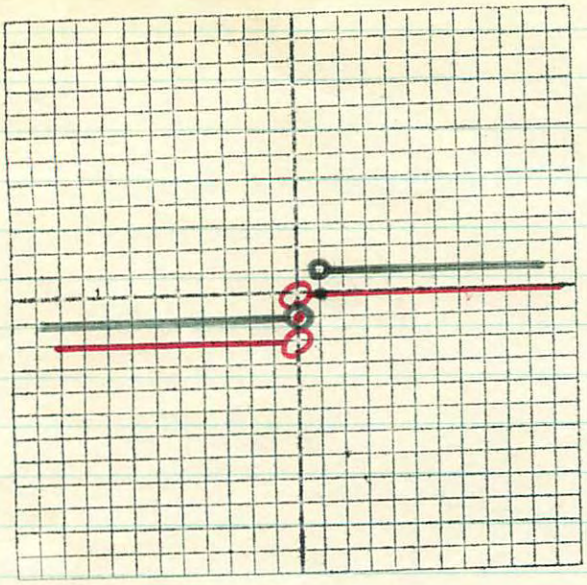


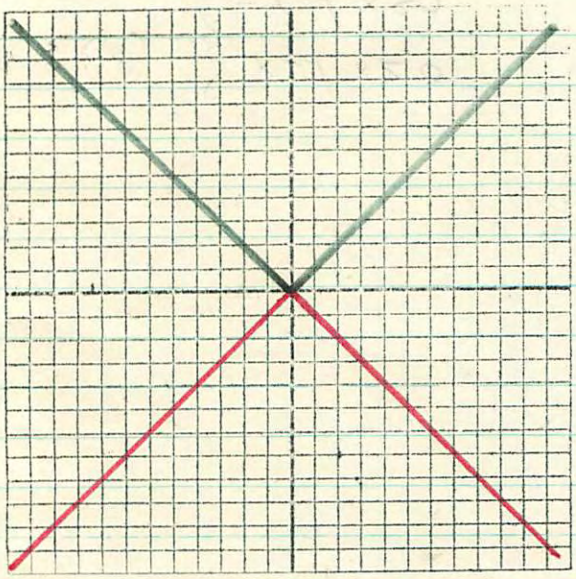
24 $g(x) \begin{cases} x \text{ if } x \geq 0 \\ -2 \text{ if } x < 0 \end{cases}$

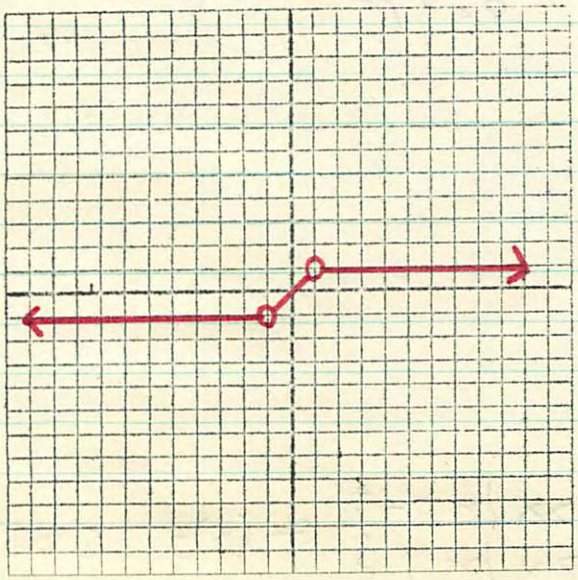


25 $h(x) \begin{cases} x \text{ if } x \geq 0 \\ -x \text{ if } x < 0 \end{cases}$



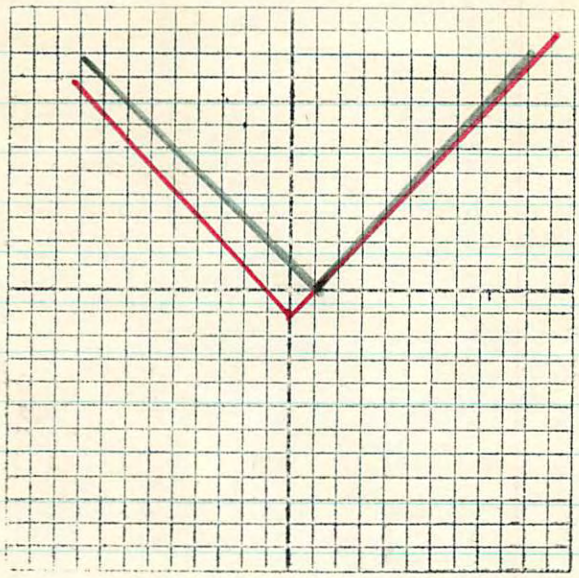






29 $x \to |x|-1$

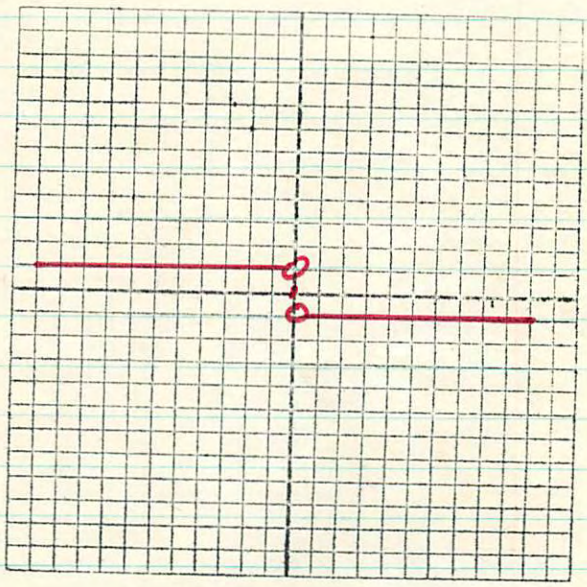
$x \to |x-1|$



30. $x \to -\operatorname{Sgn} x$

$x \to \operatorname{Sgn}(-x)$

Both the same.

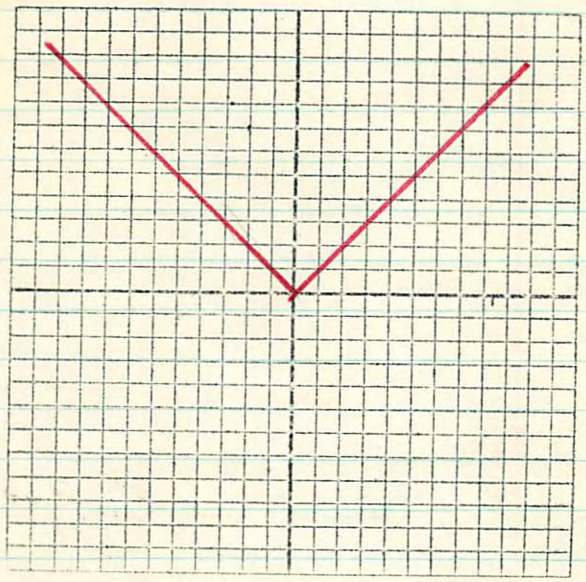


31

$x \to x \operatorname{Sgn} x$

$$x \to \begin{cases} x \text{ if } x > 0 \\ x = 0 \text{ if } x = 0 \\ -x \text{ if } x < 0 \end{cases}$$

Commonly called absolute value fctns.



32

$$\text{Si}: x \to \frac{x}{|x|}$$

if $x > 0$ $\quad \text{Si}: x = 1$

if $x < 0$ $\quad \text{Si}: x = -1$

$x = 0$ is not in Domain.

Domain of Si is $x \in \mathbb{R}$ $x \neq 0$

fctns are not equal. Sgn has 0 in Domain.

① Constant fctn odd
D→ℝ R→C horizontal line INVERSE is Relation
etc. Slope = 0 Even

Linear ② Identity fctn D→ℝ R→ℝ 45° line linear
1:1 fctn self inverse ~~Even~~ odd
It is the line and reflect fctn across it for inverse fctns

③ Absolute Value fctn D→ℝ R≥0
Consider f(x) = a|bx+c|+d as extreme case. comparable to full descriptions of Sin fctn in trig

④ Greatest integer fctn D→ℝ R⊆J
Not 1:1 Neither odd nor even

⑤ Sgn fctn D→ℝ R→{-1, 0, 1} also compares: f(x) = a sgn(bx+c)+d

Demo Piecewise fctns

|x| sgn(x) [[x]] $f(x)=\begin{cases} 2x-1 & x<2 \\ 4 & x=2 \\ x+1 & x>2 \end{cases}$

Show 1st example next page stating "This is more complicated than HW: Save the next 2 egs for Demo Next day.

$f(x) = |x+2| + \operatorname{Sgn}(x-1)$

I II III

0 0

-2 1

$$f(x) = \begin{cases} -x-3 & \text{if } x \le -2 \\ x+1 & \text{if } -2 < x < 1 \\ 3 & \text{if } x = 1 \\ x+3 & \text{if } x > 1 \end{cases}$$

I $-(x+2) - 1$

$-x-3$

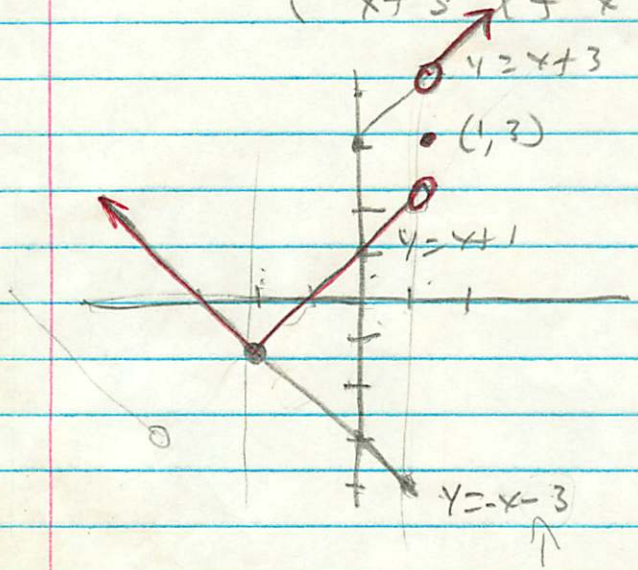
II $x+2-1$

$x+1$

$1+1=2 \neq 3$

III $x+2+1$

$x+3$



$g(x) = (\operatorname{Sgn}(x-1))^2 + \operatorname{Sgn}(x+1)$

I

0 0

-1 0 1

$$g(x) = \begin{cases} 0 & \text{if } x < -1 \\ 1 & \text{if } x = -1 \\ 2 & \text{if } -1 < x < 1 \\ 1 & \text{if } x = 1 \\ 2 & \text{if } x > 1 \end{cases}$$

I $(-1)^2 - 1$

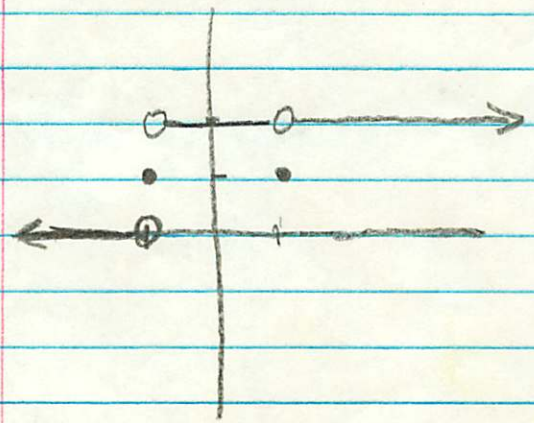
$1 - 1 = 0$

II $(-1)^2 + 1$

$1+1=2$

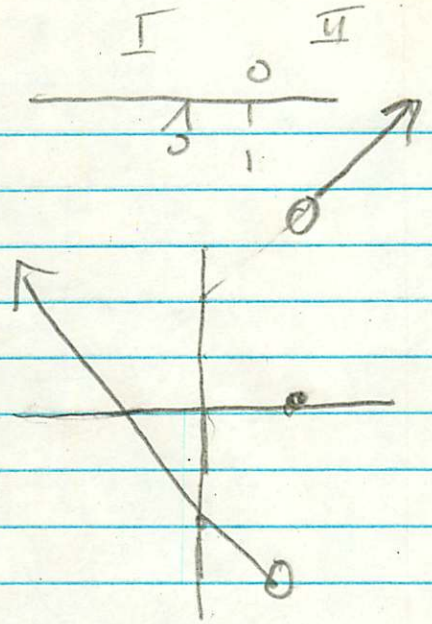
III $1^2 + 1$

2



$H(x) = (x+2)\,\mathrm{Sgn}(x-1)$

$$H(x) = \begin{cases} -x-2 & \text{if } x < 1 \\ 0 & \text{if } x = 1 \\ x+2 & \text{if } x > 1 \end{cases}$$



Given $P(x) = \frac{(x+1)(x-2)}{(x-2)(x+3)}$ $\quad Q(x) = \frac{(x-1)}{(x+1)^2(x+3)}$

$R(x) = \frac{2x+5}{x+3}$

$= \frac{x+1}{x+3}$

$D_Q \quad x \notin \{-1, -3\}$

$D_P \quad x \notin \{2, -3\}$

$A(x) = \frac{P}{Q} \quad D_A \quad x \notin \{2, -3, -1, -3, 1\}$

$D_B \quad B(x) = A(P(x))$

$\frac{x+1}{x+3} \overset{\text{set}}{=} -1$

$x+1 = -x-3$

$2x = -4$

$x = -2$

$\frac{x+1}{x+3} \overset{\text{set}}{=} -3$

$x+1 = -3x-9$

$4x = -10$

$x = -\frac{5}{2}$

$\therefore D_B \quad x \notin \{-1, -3, -2, -\frac{5}{2}\}.$

$C(x) = P(P(x))$

$\frac{x+1}{x+3} \overset{\text{set}}{=} 2$

$x+1 = 2x+6$

$-5 = x$

$\frac{x+1}{x+3} \overset{\text{set}}{=} -3$

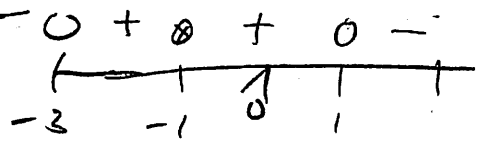
$x+1 = -3x-9$

$4x = -10$

$x = -\frac{5}{2}$

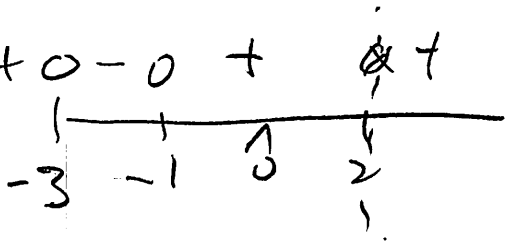
$D_C \quad x \notin \{2, -3, -5, -\frac{5}{2}\}.$

$D(x) = \sqrt{-Q(x)}$



$D_D \quad -3 < x < 1;\ x \neq -1$

$E(x) = \sqrt{P(x)}$

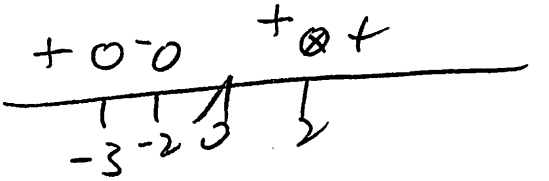


$D_E \quad x < -3 \text{ or } x > -1;\ x \neq 2$

$f(x) = \sqrt{P(x) + R(x)}$

$\frac{x+1}{x+3} + \frac{2x+5}{x+3} \geq 0$

$\frac{3x+6}{x+3}$



$D_f \quad x < -3 \text{ or } x \geq -2;\ x \neq 2$

Given $P(x) = \dfrac{x-3}{(x^2-5x+6)}$ $\quad Q(x) = \dfrac{x+2}{(x+1)(x+3)}$

$(x-3)(x-2)$

$D_Q \quad x \neq \{-3, -1\}$.

$= \dfrac{1}{x-2}$

$D_P \quad x \neq \{2, 3\}$

$A(x) = \dfrac{P}{Q}$ $\quad D_A \quad x \neq \{-3, -1, 2, 3, -2\}$ "union of restrictions" and denom $\neq 0$

"Domain of inside" Add Results of inside fctn set to restrictions of outside fctn"

$B(x) = Q(P(x))$ $\quad D_B \quad x \neq \{2, 3, \frac{5}{3}, 1\}$

$\dfrac{1}{x-2} \overset{\text{set}}{=} -3$

$1 = -3x+6$

$3x = 5$

$x = \frac{5}{3}$

$\dfrac{1}{x-2} \overset{\text{set}}{=} -1$

$1 = -x+2$

$x = 1$

$C(x) = P(P(x))$

$D_C \quad x \neq \{2, 3, \frac{5}{2}, \frac{7}{3}\}$

$\dfrac{1}{x-2} \overset{\text{set}}{=} 2$

$1 = 2x-4$

$2x = +5$

$x = +\frac{5}{2}$

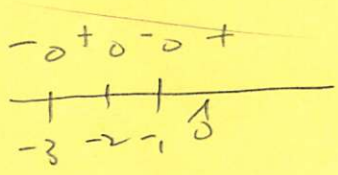
$\dfrac{1}{x-2} \overset{\text{set}}{=} 3$

$1 = 3x-6$

$3x = 7$

$x = \frac{7}{3}$

$D(x) = \sqrt{Q(x)}$



$D_D \quad -3 < x \leq -2$ or $x > -1$

$E(x) = \sqrt{P}$

$- \; 0 \; + \; \emptyset \; +$

$0 \quad 2 \quad 3$

$D_E \quad x > 2; x \neq 3$

$f(x) = \frac{1}{P+Q}$

$= \frac{1}{x-2} + \frac{x+2}{(x+1)(x+3)}$

$= \frac{x^2+4x+3+x^2-4}{(x-2)(x+1)(x+3)}$

$\frac{2x^2+4x-1}{(x-2)(x+1)(x+3)}$

$f(x) = D_f \quad \frac{(x-2)(x+1)(x+3)}{2x^2+4x-1}$ ← NOT NECESSARY. We Need to know when this is zero and that occurs when x ≠

$x = \frac{-4 \pm \sqrt{16+8}}{4}$

$\frac{-4 \pm 2\sqrt{6}}{4}$

$\frac{-2 \pm \sqrt{6}}{3}$ ←

$D_f \quad x \notin \left\{-3, -1, 2, 3, \frac{-2 \pm \sqrt{6}}{3}\right\}$

The "INVERSE method"

INVERSE fctNS

2 step ① Substitute z for x, x for $f(x)$.
② Solve for z.

z is the inverse fctn.

eg $f(x) = \frac{3x+2}{4x-1}$ D_f $x \neq \frac{1}{4}$

$x = \frac{3z+2}{4z-1}$

$4xz - x = 3z + 2$

$(4x-3)z = x+2$

$z = \frac{x+2}{4x-3}$

"By INSPECTION" the Domain of $f^{-1}(x)$ is $x \neq \frac{3}{4}$

$D_{f^{-1}}$ $x \neq \frac{3}{4}$ $\therefore$ R_f $f(x) \neq \frac{3}{4}$ $\therefore$ Thm

If $f(f^{-1}(x)) = x$ then f is 1:1.

$f\left(\frac{x+2}{4x-3}\right) = \frac{3\left(\frac{x+2}{4x-3}\right)+2}{4\left(\frac{x+2}{4x-3}\right)-1}$

$= \frac{3x+6+8x-6}{4x+8-4x+3}$

$= \frac{11x}{11} = x$ so f is 1:1

Let $z = f^{-1}$ Sub z for x and x for f(x) or y
Solve for z.

for lecture use $f(x) = \frac{x+1}{x-2}$ $x = \frac{z+1}{z-2}$ etc.

① $f(x) = x - 3$
$x = f^{-1} - 3$
$f^{-1} = x + 3$
$D_{f^{-1}} = \mathbb{R}$

② $f: x \to x + 2$
$y + 2 = x$
$y = x - 2$
$f^{-1}(x) = x - 2$
$D_{f^{-1}} = \mathbb{R}$

③ $f(x) = 2x$
$x = 2y$
$y = \frac{1}{2}x$
$D_{f^{-1}} = \mathbb{R}$

④ $f(x) = -2x$
$x = -2y$
$y = -\frac{1}{2}x$
$D_{f^{-1}} = \mathbb{R}$

⑤ $f(x) = 2x - 1$
$x = 2y - 1$
$\frac{1}{2}(x+1) = y$
$D_{f^{-1}} = \mathbb{R}$

⑥ $f(x) = 2x + 3$
$x = 2y + 3$
$\frac{1}{2}(x-3) = y$
$D_{f^{-1}} = \mathbb{R}$

⑦ $f(x) = \frac{1}{x}$
$x = \frac{1}{y}$
$y = \frac{1}{x}$
$D_{f^{-1}} = x \neq 0$

⑧ $f(x) = \frac{1}{x-1}$
$x = \frac{1}{y-1}$
$y - 1 = \frac{1}{x}$
$y = \frac{1+y}{x}$ $x \neq 0$

⑨ $f(x) = \frac{1}{2x+1}$
$x = \frac{1}{2y+1}$
$2y + 1 = \frac{1}{x}$
$y = \frac{1-x}{2x}$
$x \neq 0$

⑩ $f(x) = \frac{x}{x+1}$
$x = \frac{y}{y+1}$
$xy + x = y$
$x = y - xy$
$x = y(1-x)$
$y = \frac{x}{1-x}$
$x \neq +1$

(11) $f(x) \to \frac{x}{x-1}$

$x = \frac{y}{y-1}$

$xy - x = y$

$y(x-1) = x$

$y = \frac{x}{x-1}$

$x \neq 1$

(12) $f(x) = x^3$

$x = y^3$

$y = \sqrt[3]{y}$

(13) $f(x) = (x-2)^3$

$x = (y-2)^3$

$\sqrt[3]{x} = y-2$

$y = 2 + \sqrt[3]{x}$

(14) $f(x) = x^3 - 2$

$x = y^3 - 2$

$\sqrt[3]{(x+2)} = y$

(15) $f(x) = x^{\frac{1}{3}}$

$x = y^{\frac{1}{3}}$

$x^3 = y$

(16) $f(x) = (x-2)^{\frac{1}{3}}$

$x = (y-2)^{\frac{1}{3}}$

$x^3 + 2 = y$

(17) $f(x) = x^{\frac{1}{3}} - 2$

$x = y^{\frac{1}{3}} - 2$

$(x+2)^3 = y$

(18) $f(x) = \frac{1}{x^3}$

$x = \frac{1}{y^3}$

$y^3 = \frac{1}{x}$

$y = \sqrt[3]{\frac{1}{x}}$

(19) $f(x) = \frac{5}{x}$

$x = \frac{5}{y}$

$y = \frac{5}{x}$

$x \neq 0$

(20) $f(x) = \frac{1}{x+2}$

$x = \frac{1}{y+2}$

$xy + 2x = 1$

$y = \frac{1-2x}{x}$

$x \neq 0$

(21) $f(x) = \frac{5}{x+2}$

$x = \frac{5}{y+2}$

$xy + 2x = 5$

$y = \frac{5-2x}{x}$

$x \neq 0$

22 $f(x) = \frac{x}{x+c}$

$x = \frac{y}{y+c}$

$xy + xc = y$

$xc = y(1-x)$

$\frac{xc}{1-x} = y$

$x \neq 1$

23 $f(x) \to \frac{x+b}{x}$

$x = \frac{y+b}{y}$

$xy - y = b$

$y(x-1) = b$

$y = \frac{b}{x-1} \quad x \neq 1$

24 $f(x) = \frac{ax+b}{x}$

$x = \frac{ay+b}{y}$

$xy = ay + b$

$y(x-a) = b$

$y = \frac{b}{x-a}$

where does this come from?

↓

$a^2 + bc \neq 0$

25 $f(x) = \frac{ax+b}{cx-a}$

$x = \frac{ay+b}{cy-a}$

$cyx - ax = ay + b$

$y(cx-a) = b + ax$

$y = \frac{b+ax}{cx-a}$

(27) $a^2 + bc \neq 0 \Rightarrow bc = -a^2 \Rightarrow c = \frac{-a^2}{b}$

$f(x) = \frac{ax+b}{cx-a} = \frac{ax+b}{\frac{-a^2x - ab}{b}} = \frac{ax+b}{\frac{-a(ax+b)}{b}} = \frac{b(ax+b)}{-a(ax+b)} = \frac{b}{-a}$

$\therefore f(x) = \frac{b}{-a}$ is a constant fctn and f would not be 1:1 $\therefore$ no f^{-1}

24 $f(x) \to \frac{ax+b}{cx+d}$ $ad-bc \neq 0$ ← This comes from $\frac{ax+b}{cx+d}$

$ad-bc$: this is similar to evaluating a 2x2 determinant. (important!!!!!)

$$y = \frac{ax+b}{cx+d} \Rightarrow x = \frac{ay+b}{cy+d} =$$

$$cxy + dx = ay + b$$

$$cxy - ay = b - dx$$

$$y(cx-a) = b-dx$$

(28) $$y = \frac{b-dx}{cx-a} = f^{-1}$$

(29)
$$\frac{a\left(\frac{b-dx}{cx-a}\right)+b}{c\left(\frac{b-dx}{cx-a}\right)+d} = \frac{\frac{ab-adx+bcx-db}{cx-a}}{\frac{cb-cdx+cdx-ad}{cx-a}}$$

$$\frac{bcx-adx}{bc-ad} = \frac{x(bc-ad)}{(bc-ad)} = x$$

(30)
$$ad-bc=0$$
$$ad=bc$$
$$a=\frac{bc}{d}$$

$$f(x) = \frac{\frac{bcx+b}{d}}{cx+d}$$

$$\frac{\frac{bcx+bd}{d}}{cx+d} =$$

$$\frac{\frac{b(cx+d)}{d}}{(cx+d)}$$

$$f(x) = \frac{b}{d}$$

agn. $f(x)$ would be a constant fctn.

Note: 25-30. These describe how to find if a fctn is 1:1. ie if it is $f(f^{-1}) = f^{-1}(f) = I$.

$$f(x) = \frac{ax+b}{cx+d}$$

f will not have inverse fctn whenever f = constant,

when would f be a constant? what constant would it be?

① f would be a constant if $b = d = 0$; then $f = \frac{a}{c}$

$$\frac{ax+b}{cx+d} \overset{\text{set}}{=} \frac{a}{c}$$

$$acx + bc = acx + ad$$

$ad - bc = 0$ this relationship would exist if f were a constant.

② f would be a constant if $a = c = 0$; then $f = \frac{b}{d}$

$$\frac{ax+b}{cx+d} \overset{\text{set}}{=} \frac{b}{d}$$

$$adx + bd = bcx + bd$$

$(ad - bc)x = 0 \Rightarrow ad - bc = 0$ or $x = 0$,

Since x is a variable it may = 0 when x = 0 but it would change so the situation would become when $ad - bc = 0$

which are the same conditions as in 2 above.

③ Do there any other values of the constants so that f would be a constant? No. In order to be a constant fctn the "x" must reduce off as in ① or must not be there in the 1st place as in ②.

Do Not Write on This Paper!

10 Minutes
Quiz #1

Determine The Domain

$$A(x) = \sqrt{\frac{(x-1)^2(x+2)}{x-3}}$$

$$B(x) = \sqrt{\frac{-(x+2)(x+3)}{(x+3)(x+4)}}$$

$$C(x) = \frac{x\sqrt{x}}{x^2+5x+6}$$

$$D(x) = \sqrt{1-\frac{1}{x}}$$

Analysis Chpt 1

Quiz #1

Within 10 minutes

Given Day HW
Pg 45-46 1-23 odd
25-27
→ collected

Determine The Domain

$$A(x) = \sqrt{\frac{(x-1)^2(x+2)}{x-3}}$$

$+ \; 0 \; - \; \otimes \; - \; 0 \; +$
$-2 \quad 0 \quad 1 \quad 3$

D_A $x \le -2$ or $x = 1$ or $x > 3$

$$B(x) = \sqrt{\frac{-(x+2)(x+3)}{(x+3)(x+4)}}$$

$- \; 0 \; + \; \otimes \; + \; 0 \; -$
$-4 \quad -3 \quad -2 \quad 0$

D_B $-4 < x \le -2$; $x \neq -3$

$$C(x) = \frac{x\sqrt{x}}{x^2+5x+6}$$

$- \; 0 \; +$
0

D_C $x \ge 0$

if they include $x \neq \{-2, -3\}$ wrong! They don't show understanding of concept.

$$D(x) = \sqrt{1 - \frac{1}{x}}$$

$$\sqrt{\frac{x-1}{x}}$$

$+ \; 0 \; - \; 0 \; +$
$0 \quad 1$

D_D $x < 0$ or $x \ge 1$

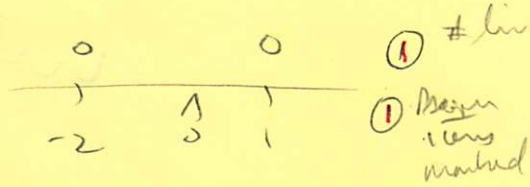
4	+
3	c
2	0
1	−
0	=

Ark ch 1

12 minutes of the writing Quiz on chalkboard. Quiz #2

WRITE as piecewise fctns. Graph. Specifically. State the range.

10pts $A(x) = |x+2| \operatorname{sgn}(x-1)$

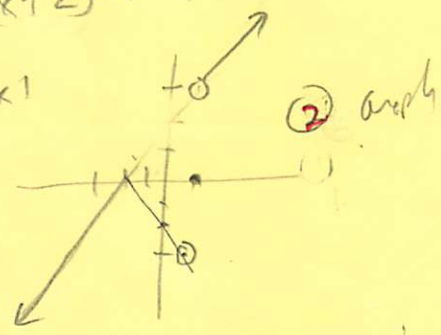


① # lin
① Diagram items marked.

$$A(x) = \begin{cases} x+2 & \text{if } x < -2 \\ -x-2 & \text{if } -2 \le x < 1 \\ 0 & \text{if } x = 1 \\ x+2 & \text{if } x > 1 \end{cases}$$

4 terms ①

$-(x+2)(-1)$
$(x+2)(-1)$

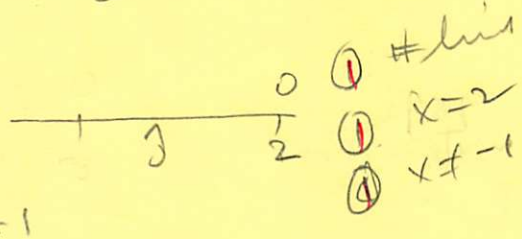


② graph

R_A $A(x) \le 0$ or $A(x) > 3$ ①

10pts $B(x) = (x+1) + \operatorname{sgn}(2-x)$

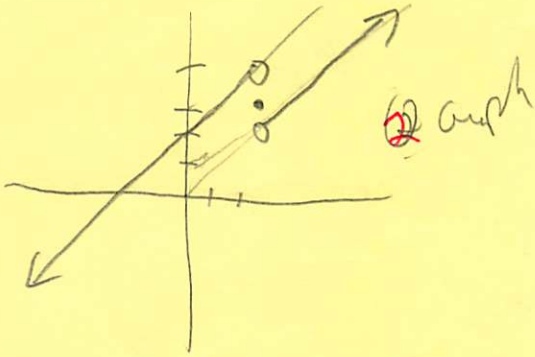
3 terms ①



① # lin
① x=2
① x≠-1

$$B(x) = \begin{cases} x+2 & \text{if } x < 2 \\ 3 & \text{if } x = 2 \\ x & \text{if } x > 2 \end{cases}$$

x+1 +1



② graph

R_B $B(x) \in \mathbb{R}$. ①

15 minutes.

19, 20 +
16, 17, 18 ✓
13, 14, 15 0
8 – 12 –
0 – 7 =

Chop Quiz #3
into Quiz 3 & Quiz 4,
it will require less
time in one shot.

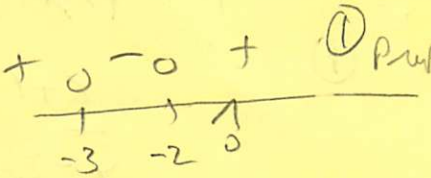
AWL
Ch 1

Quiz #3

allow 15 min after writing Quiz on board.

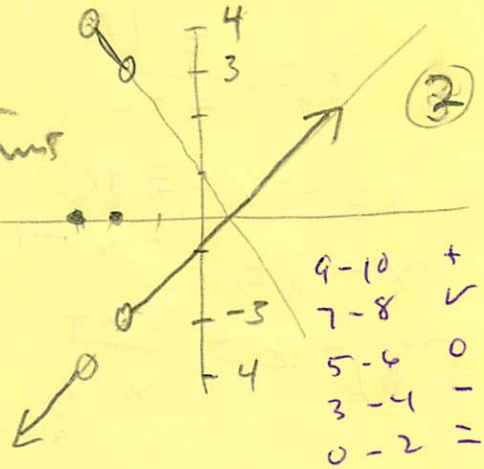
I Re write as a piecewise fctn. Graph. Specifically (10 pts) state the range

$$A(x) = (x-1)\,\mathrm{Sgn}\left[\frac{x^2+5x+6}{(x+2)(x+3)}\right]$$



$$A(x) = \begin{cases} x-1 & \text{① if } x < -3 \\ 0 & \text{① if } x = -3 \\ -x+1 & \text{① if } -3 < x < -2 \\ 0 & \text{① if } x = -2 \\ x-1 & \text{① if } x > -2 \end{cases}$$

5 terms ①



9-10 +
7-8 ✓
5-6 0
3-4 −
0-2 =

R_A $A(x) < -4$ ① or $A(x) > -3$

II

2 pts ea.

$$f(x) = \frac{(x+1)(x+1)}{(x+1)(x-2)} \qquad g(x) = \frac{(x-2)(x+3)}{(x+3)(x-4)}$$

written IPI Properly for abc Actual answer IPT

A) find the Domain of $A(x) = f(x)\,g(x)$

D_A $x \neq \{-1, 2, -3, 4\}$

B) find the Domain of $B(x) = f(g(x))$

D_B $x \neq \{-3, 3, 4, 6\}$

$\frac{x-2}{x-4} \overset{\text{set}}{=} -1$ $\quad x-2 = -x+4$ $\quad 2x = 6$ $\quad x = 3$

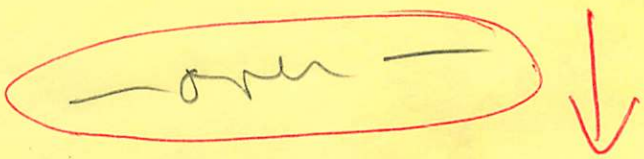
$\frac{x-2}{x-4} \overset{\text{set}}{=} 2$ $\quad x-2 = 2x-8$ $\quad 6 = x$

C) find the Domain of $C(x) = g(f(x))$

D_C $x \neq \{-1, 2, \frac{5}{4}, 3\}$

$\frac{x+1}{x-2} \overset{\text{set}}{=} -3$ $\quad x+1 = -3x+6$ $\quad 4x = 5$ $\quad x = \frac{5}{4}$

$\frac{x+1}{x-2} \overset{\text{set}}{=} 4$ $\quad x+1 = 4x-8$ $\quad 9 = 3x$ $\quad x = 3$



5) find $f^{-1}(x)$

$f(x) = \frac{(x+1)(x+1)}{(x-1)(x-2)}$

$\frac{x+1}{x-2}$

$x = \frac{z+1}{z-2}$

$xz - 2x = z + 1$

$(x-1)z = 2x+1$

$f^{-1}(x) = z = \frac{2x+1}{x-1}$ +p1

6) show $f^{-1}(f(x)) = x$

$\frac{\frac{2x+1}{x-1} + 1}{\frac{2x+1}{x-1} - 2}$

$\frac{2x+1+x-1}{2x+1-2x+2}$

$\frac{3x}{3}$

x

Score
18-20 +
14-17 ✓
10-13 0
6-9 −
0-5 =

Quiz #2 make up

Rewrite as Piece-wise fctn.

Draw the graph.

State the Range

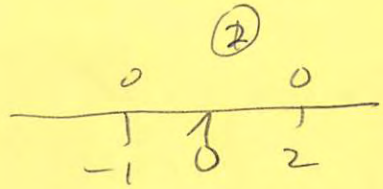
DO NOT WRITE on this Paper!

$$A(x) = |2-x| \operatorname{sgn}(1+x)$$

$$B(x) = |2-x| + \operatorname{sgn}(1-x)$$

Quiz #2 make up.

$A(x) = |2-x| \operatorname{sgn}(1+x)$



$$A(x) = \begin{cases} x-2 & x<-1 \\ 0 & x=-1 \\ 2-x & -1<x<2 \\ x-2 & x\geq 2 \end{cases} \quad ③$$

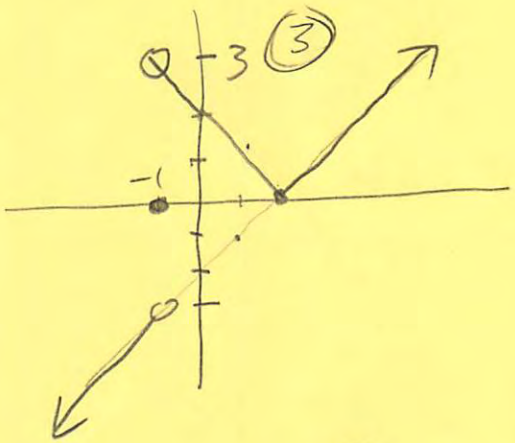
$(2-x)(-1)$

$(2-x)(1)$

$-(2<x)(1)$

R_A $A(x)<-3$ or

② $A(x)\geq 0$



$B(x) = |2-x| + \operatorname{sgn}(1-x)$

$$B(x) = \begin{cases} 3-x & x<1 \\ 1 & x=1 \\ 1-x & 1<x<2 \\ -3+x & x\geq 2 \end{cases} \quad ③$$

$(2-x)+1$

$2-x-1$

$-(2-x)-1$

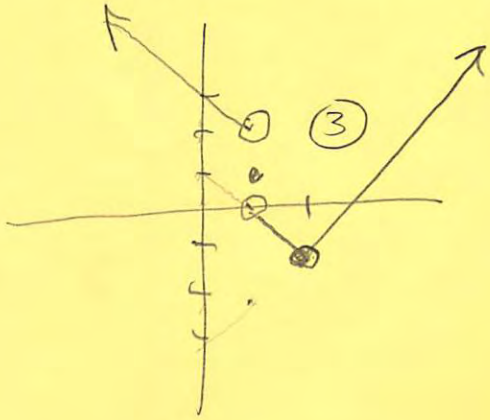
R_B $B(x)<0$

or

② $B(x)=1$

or

$B(x)>2$



① $f=\{(1,2),(2,4),(3,6)\}$ $g=\{(1,1),(2,2),(3,2)\}$

$D_f=\{1,2,3\}$ $D_g=\{1,2,3\}$

$\therefore D_{f+g}=D_{fg}=D_{\frac{f}{g}}=\{1,2,3\}$

$f+g=\{(1,3),(2,6),(3,8)\}$

$f-g=\{(1,1),(2,2),(3,4)\}$

$fg=\{(1,2),(2,8),(3,12)\}$

$\frac{f}{g}=\{(1,2),(2,2),(3,3)\}$

$f^2=\{(1,4),(2,16),(3,36)\}$

$g^2=\{(1,1),(2,4),(3,4)\}$

④ $f=\{(1,0),(2,-4),(3,5),(4,1)\}$

$g=\{(1,0),(2,2),(3,0),(4,2),(5,3)\}$

$f+g=\{(1,0),(2,-2),(3,5),(4,3)\}$

$f-g=\{(1,0),(2,-6),(3,5),(4,-1)\}$

$fg=\{(1,0),(2,-8),(3,0),(4,2)\}$

$\frac{f}{g}=\{(2,-2),(4,\frac{1}{2})\}$

(9) $f: x \to 6x \quad g: x \to 2$

$f+g = 6x+2$
$fg = 12x$
$\frac{f}{g} = \frac{6x}{2} = 3x$
$\Big\} D \Rightarrow \mathbb{R}$

(10) $f(x) \to 2 \quad g: x \to x$

$f+g = x+2$
$fg = 2x$
$\Big\} D \Rightarrow \mathbb{R}$
$\frac{f}{g} = \frac{2}{x} \quad \mathbb{R}, x \neq 0$

(11) $f: x \to x \quad g(x) \to x^2+1$

$f+g = x^2+1+x$
$fg = x^3+x$
$\frac{f}{g} = \frac{x}{x^2+1}$
$\Big\} \mathbb{R}$

(12) $f(x) \to x^2 \quad g = I$

$f+g = x^2+x$
$f(g) = x^3$
$\Big\} \mathbb{R}$
$\frac{f}{g} = \frac{x^2}{x} = x \quad x \neq 0$

(13) $f(x) = x^2 \quad g(x) = \frac{1}{x}$

$f+g = x^2 + \frac{1}{x} = \frac{x^3+1}{x} \quad x \neq 0$
$f(g) = x^2 \quad \frac{1}{x} = x \quad x \neq 0$
$\frac{f}{g} = \frac{x^2}{\frac{1}{x}} = x^3 \quad x \neq 0$

(14) $f(x) \to x-1 \quad g(x) \to x^2-1$

$f+g = x^2+x-2$
$g(g) = x^3-x^2-x+1$ } $\mathbb{R}$

$\frac{f}{g} = \frac{x-1}{x^2-1} = \frac{1}{x+1} \quad x \neq \pm 1$

(15) $f(x) = x^2 \quad g = \text{Sgn}$

$f+g = \begin{cases} x^2+1 & x>0 \\ x^2 & x=0 \\ x^2-1 & x<0 \end{cases}$ $\mathbb{R}$ or 0, $x=0$

$fg = \begin{cases} x^2 & x>0 \\ 0 & x=0 \\ -x^2 & x<0 \end{cases}$ $\mathbb{R}$

$\frac{f}{g} = \begin{cases} x^2 & x>0 \\ -x^2 & x<0 \end{cases}$ $x \neq 0$

(16) $f(x) = |x| \quad g = I$

$f+g = \begin{cases} x+x=2x & x \geq 0 \\ -x+x=0 & x<0 \end{cases}$ $\mathbb{R}$ Note $-x$ is >0

$fg = \begin{cases} x^2 & x \geq 0 \\ -x^2 & x<0 \end{cases}$

$\frac{f}{g} = \begin{cases} 1 & x>0 \\ -1 & x<0 \end{cases}$ $x \neq 0$

ORAL EX.

① $D_{g\circ f}$ $\{1,2,3\}$ $\{(1,2),(2,3),(3,1)\}$

② $D_{g\circ f}$ $\{1,2,3,4\}$ $\{(1,10),(2,10),(3,11),(4,10)\}$

③ $D_{g\circ f}$ $\{1,2,3,4,5\}$ $\{(1,1),(2,2),(3,3),(4,4),(5,5)\}$.

④ $D_{g\circ f}$ $\{2,3,4\}$ $\{(2,13),(3,14),(4,12)\}$

⑤ $D_{g\circ f} = \{1,2,3\}$ $\{(1,6),(2,5),(3,4)\}$

⑥ $D_{g\circ f} = \{1,2,3\}$ $\{(1,4),(2,5),(3,6)\}$

⑪ $f: x \to x^2$ $g: x \to x+1$

$D_f = \mathbb{R} = D_g$ Thm No restrictions on D_g ; none on $g\circ f$.

$g\circ f = x^2+1$

⑫ $f(x) = x+1$ $g(x) = x^2$ $D_{g\circ f} = \mathbb{R}$

$g\circ f = (x+1)^2 = x^2+2x+1$

⑬ $f(x) = x-1$ $g(x) = \frac{1}{x}$ $f(x) \neq 0 \Rightarrow x \neq 1$

Notice we do let $x=0$

$g\circ f = \frac{1}{x-1}$

⑭ $f(x) = \frac{1}{x}$ $g(x) = x-1$ $D_f \Rightarrow x \neq 0$

$g\circ f = \frac{1}{x} - 1 = \frac{1-x}{x}$

⑮ $f(x) = \sqrt{x}$ $g(x) \Rightarrow x^2$ D_f $x \geq 0$

$g\circ f = x$ $x \geq 0$

⑯ $f(x) = x+1$ $g(x) = x-1$

$g\circ f = x$ $D_{g\circ f} \Rightarrow \mathbb{R}$

Written Ex.

① $f(x) = 2x \quad g(x) = x^2 + 4$

$g \circ f = 4x^2 + 4 \quad D_{g \circ f}$ $\mathbb{R}$

② $f(x) = x^2 + 4 \quad g(x) = 2x$

$g \circ f = 2(x^2+4) = 2x^2 + 8 \quad D$ $\mathbb{R}$

③ $f(x) \to x - 1 \quad g(x) \to \sqrt{x} \quad D \; x \geq 1$

$g \circ f \; \sqrt{x-1}$

④ $f(x) = \sqrt{x} \quad g(x) = x - 1 \quad x \geq 0$

$g \circ f = \sqrt{x} - 1$

* ⑤ $f(x) = x^2 \quad g(x) = \sqrt{x} \quad D = \mathbb{R}$

$g \circ f = |x|$ ← important pblm.

⑥ $f(x) = 2x \quad g(x) = \sqrt{4 - x^2} \quad D_g \; |x| \leq 2$

$g \circ f = \sqrt{4 - 4x^2} = 2\sqrt{1 - x^2} \quad |2x| \leq 2$

$|x| \leq 1 \quad D_{g \circ f}$

⑦ $f(x) = -x^2 \quad g(x) = \sqrt{x}$

$D_{g \circ f} = \{0\}$

$g \circ f = \sqrt{-x^2}$

⑧ $f(x) = \frac{1}{x} \quad g(x) = x^2 - 1 \quad x \neq 0$

$g \circ f \; \frac{1 - x^2}{x^2}$

(9) $f(x) = x^2 - 1 \quad g(x) = \frac{1}{x}$ $\quad D\ x \neq \pm 1$

$g \circ f = \frac{1}{x^2-1}$

(10) $f(x) = x \quad g(x) = x^2 + 2 \quad D\ \mathbb{R}$

$g \circ f = x^2 + 2$ Notice $f(x)$ is I

(11) $f(x) = x^2 + 2 \quad g(x) = x \quad D = \mathbb{R}$

g is I.

$g \circ f = x^2 + 2$

(12) $f(x) = |x| \quad g(x)\ x+2 \quad D = \mathbb{R}$

$g \circ f = |x| + 2$

(13) $f(x) = x+2 \quad g(x) = |x| \quad D = \mathbb{R}$

$g \circ f = |x+2|$

(14) $f(x) = |x| \quad g(x) = Sgn\ x \quad D = \mathbb{R}$

$g \circ f = Sgn\ |x| = \begin{cases} 0 & \text{if } x = 0 \\ 1 & \text{if } x \neq 0 \end{cases}$

(15) $f(x) = Sgn\ x \quad g(x) = |x|$

$g \circ f = |Sgn\ x| = \begin{cases} 0 & \text{if } x = 0 \\ 1 & \text{if } x \neq 0 \end{cases}$

(16) $f(x) = x^3 \quad g(x) = \sqrt[3]{x} \quad D = \mathbb{R}$

$g \circ f = x$

17 $f(x)=\frac{1}{x}$ $g(x)=\frac{1}{x}$ $x\neq 0$

$g\circ f = x$

18 $f(x)=\frac{1}{x-1}$ $g(x)=\frac{x+1}{x}$ $x=0 \Rightarrow$

$x\neq 1$

$$g\circ f = \frac{\frac{1}{x-1}+1}{\frac{1}{x-1}}$$

$$= \frac{\frac{1+x-1}{x-1}}{\frac{1}{x-1}} = \frac{\frac{x}{x-1}}{\frac{1}{x-1}} = x \quad x\neq 1$$

19

$f(x)=\frac{x+1}{x}$ $g(x)=\frac{1}{x-1}$

$$g\circ f = \frac{1}{\frac{x+1}{x}-1} = \frac{1}{\frac{x+1-x}{x}} = \frac{1}{\frac{1}{x}} = x \quad x\neq 0$$

$f(x)\neq 1 \Rightarrow \frac{x+1}{x}=1$

$x+1=x$

20 $f(x)=1$ $g(x)=x^2-1$

$g\circ f = g(1) = 0$ $x\in\mathbb{R}$

(21) $\Phi(x) = (x+1)^5$ let $f(x) = x+1$ and $g(x) = x^5$

then $\Phi(x) = g \circ f$

(22) $\Phi(x) = (x^2+1)^5$ let $f(x) = x^2+1$, let $g(x) = x^5$

(23) $\Phi(x) = \sqrt{x^2+4}$ let $f(x) = x^2+4$, $g(x) = \sqrt{x}$

(24) $\Phi(x) = \frac{1}{x^2+1}$ let $f(x) = x^2+1$, $g(x) = \frac{1}{x}$

(25) $\Phi(x) = \frac{1}{x^2-1}$ let $f(x) = x^2-1$, $g(x) = \frac{1}{x}$

(26) $\Phi(x) = \sqrt{1-x^2}$ let $f(x) = 1-x^2$, $g(x) = \sqrt{x}$

(27) $h \circ (g \circ f) = (h \circ g) \circ f$ $f(x) = x^2$ $g(x) = x+1$, $h(x) = \frac{1}{x}$

$g \circ f = g(x^2) = x^2+1$

$h \circ (g \circ f) = h(x^2+1) = \frac{1}{x^2+1}$

$h \circ g = h(x+1) = \frac{1}{x+1}$

$(h \circ g) \circ f = \frac{1}{x^2+1}$

Use this for HW

On overhead

Given $h(x) = f \circ g(x)$, DETERMINE:

D_f, R_f, D_g, R_g, D_h, R_h for EACH:

① $f(x) = \sqrt{x-4}$ $\quad g(x) = x+4$

② $f(x) = \sqrt{x-4}$ $\quad g(x) = (x+4)^2$

③ $f(x) = x+4$ $\quad g(x) = \sqrt{x-4}$

④ $f(x) = -\sqrt{x}$ $\quad g(x) = -\sqrt{4-x}$

⑤ $f(x) = \log x$ $\quad g(x) = \frac{1}{4-x^2}$

⑥ $f(x) = \sqrt{x-1}$ $\quad g(x) = \frac{1}{x}$

① $f(x)=\sqrt{x-4}$ $g(x)=x+4$

$h(x)=\sqrt{x+4-4}=\sqrt{x}$

D_f $x\geq 4$
R_f $f(x)\geq 0$
D_g $x\in\mathbb{R}$
R_g $g(x)\in\mathbb{R}$.
D_h $x\in R$ $x\geq 0$
R_h $h(x)\geq 0$

② $f(x)=x+4$ $g(x)=\sqrt{x-4}$

$h(x)=4+\sqrt{x-4}$

$\sqrt{x^2+8x+12}$

D_g $x\in\mathbb{R}$
R_g $g(x)\geq 0$
D_f $x\geq +4$
R_f $f(x)\geq 0$
D_h $x\geq +4$.
R_h $h(x)\geq 4$.

③ $f(x)=\sqrt{x-4}$ $g(x)=(x+4)^2$

$(x+4)^2\geq 4$
$x^2+8x+16\geq 4$
$x^2+8x+12\geq 0$
$(x+6)(x+2)\geq 0$

$h(x)=$ $x^2+4x+12$

Set up with $x\in\mathbb{R}$ also. See if mean is noticed.

If >0 $x+6\geq 0$ & $x+2\geq 0$ or $x+6<0$ & $x+2<0$
$x\geq -6$ $x\geq -2$ $x\leq -6$ $x<-2$

D_f $x\geq 4$
R_f $f(x)\geq 0$
D_g $x\in\mathbb{R}$
R_g $g(x)\geq 0$.
D_h $x\leq -6$ or $x\geq -2$
R_h $h(x)\geq 0$

$h(x)=\sqrt{(x+4)^2-4}=\sqrt{x^2+8x+12}$

④ $f(x)=-\sqrt{x}$ $g(x)=-\sqrt{4-x}$.

$h(x)=f(-\sqrt{4-x})=-\sqrt{-\sqrt{4-x}}$

D_f $x\geq 0$
R_f $f(x)\leq 0$
D_g $x\leq 4$
R_g $g(x)\leq 0$.
D_h $x=4$.
R_h $h(x)=0$

Ditto continued.

⑤ $f(x) = \log x \qquad g(x) = \frac{1}{4-x^2}$

D_f $x > 0$
R_f $f(x) \in \mathbb{R}$.
D_g $\mathbb{R}$ $x \neq \pm 2$
R_g $g(x) \in \mathbb{R}\ g(x) \neq 0$
D_h $|x| < 2$
R_h $\mathbb{R}$.

$x = \frac{1}{4-y^2}$

$4x - 4y^2 = 1$

$-4y^2 = 1 - 4x$

$y^2 = \frac{4x-1}{4}$

$y = \pm\frac{\sqrt{2x-1}}{2}$

$\frac{1}{4-x^2} > 0$

$1 > 0$

$f(g(x)) = f\left(\frac{1}{4-x^2}\right) = \log\left(\frac{1}{4-x^2}\right)$

$\frac{1}{(2-x)(2+x)} > 0$

$2-x>0 \quad 2+x>0$ or $2-x<0 \quad 2+x<0$

$x-2<0 \quad x>-2$ | $x-2>0 \quad x<-2$

$x<2$ | $x>2$

⑥ $f(x) = \sqrt{x-1} \qquad g(x) = \frac{1}{x}$

D_f $x \geq 1$
R_f $f(x) \geq 0$
D_g $x \in \mathbb{R}, x \neq 0$
R_g $g(x) \in \mathbb{R}\ x \neq 0$
D_h $0 < x \leq 1$
R_h $h(x) \geq 0$

$h(x) = \sqrt{\frac{1}{x} - 1}$

$\sqrt{\frac{1-x}{x}}$

set up until $x \geq 1$ & see if error is noticed.

$\frac{1}{x} \geq 1$

$1 \geq x$

$\frac{1}{x} - 1 > 0$

$\frac{1}{x} \geq 1$ if $x > 0$ if $x < 0$

use as during Carrying pblm. list $D_{f^{-1}}$ and $R_{f^{-1}}$

oral ex.

① $f = \{(0,0), (1,2), (2,4)\}$ 1:1

$D_{f^{-1}} = \{0, 2, 4\}$ $R_{f^{-1}} = \{0, 1, 2\}$

② $f = \{(0,1), (1,2), (2,1)\}$ NOT 1:1

③ $f = \{(-1,0), (2,-1), (0,0)\}$ Not 1:1

④ $f = \{(-1,1), (0,0), (1,-1)$ 1:1

$D_{f^{-1}} = \{1, 0, -1\}$ $R_{f^{-1}} = \{-1, 0, 1\}$

⑤ $f = \{(1,1), (2, \frac{1}{2}), (3, \frac{1}{3})\}$ 1-1

$D_{f^{-1}} = \{1, \frac{1}{2}, \frac{1}{3}\}$ $R_{f^{-1}} = \{1, 2, 3\}$

⑥ $f = \{(x, 2x) : x \in \{-1, 0, 1\}\}$ 1-1

$D_{f^{-1}} = \{-2, 0, 2\}$ $R_{f^{-1}} = \{-1, 0, 1\}$

⑦ $f = \{(x, x^2) \mid x \in (-1, 0, 1\}$ NO

⑧ $f = \{(x, x^2) \; x \in \{0, 1, 2\}$ 1-1

$D_{f^{-1}} = \{0, 1, 4\}$ $R_{f^{-1}} = \{0, 1, 2\}$

⑨ $f = \{(x, |x|) \; x \in \{-2, 1, 3\}\}$ 1-1

$D_{f^{-1}} = \{2, 1, 3\}$ $R_{f^{-1}} = \{-2, 1, 3\}$

⑩ $f = \{(x, \frac{1}{x}) : x \in \{-2, -1, 1, 2\}$ 1-1

$D_{f^{-1}} = \{-\frac{1}{2}, -1, 1, 2\}$ $R_{f^{-1}} = \{-2, -1, 1, 2\}$.

ANALYSIS
mumaugh

① For the time being, we will work with Real Numbers ONLY

② $f: X \to X^2$ means $f(x) = x^2$

The word "mapping" and "transformation" are used sometimes to mean function. Thus we may say "f maps x into x^2"; or f maps 3 into 9. In this example "9" is the image of "3"; "3" is the "pre-image" of "9."

③ Given a fctn f, $f(x) = \frac{1}{x}$

A) Find D_f & R_f

B) We would like to graph "analytically" instead of the "plot the points" method.

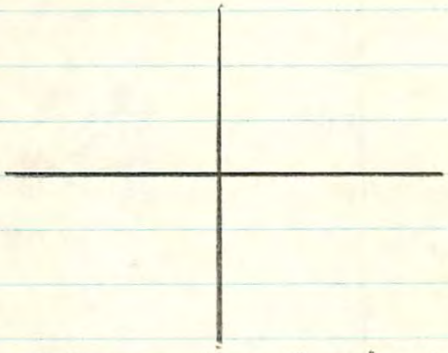
C) Consider:

I if $x > 0$, $f(x)$ ___ 0.
This means 1st Quadrant.
as $x \to +\infty$, $f(x) \to$ ___.
as $x \to 0^+$, $f(x) \to$ ___.

II if $x < 0$, $f(x)$ ___ 0.
This means ___ Quadrant
as $x \to -\infty$, $f(x) \to$ ___.
as $x \to 0^-$, $f(x) \to$ ___.

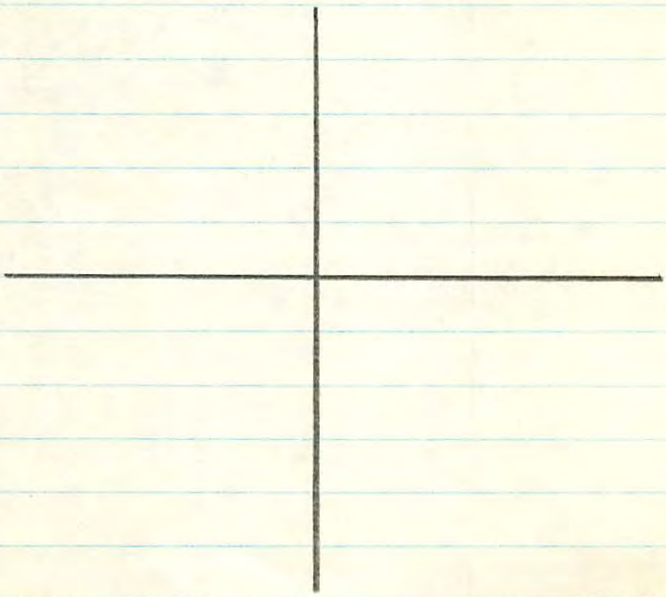
III Notice $f(1) = 1$
$f(1+\varepsilon)$ ___ 1
$f(1-\varepsilon)$ ___ 1

Sketch $f(x) = \frac{1}{x}$ using the Determined information.

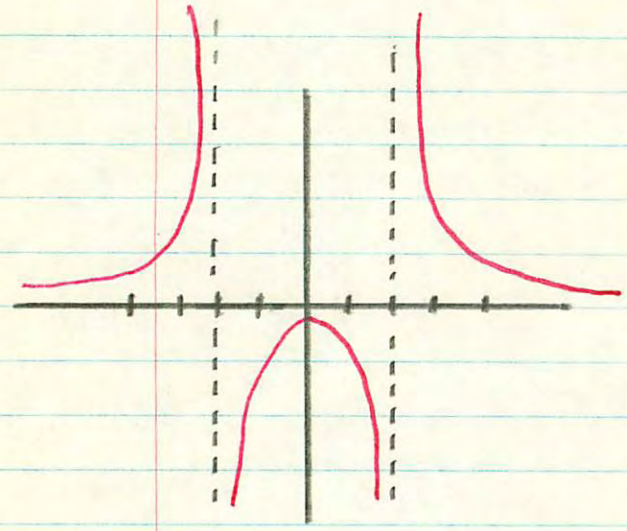


④ Do likewise for $h(x) = -\frac{2}{x}$

List comments here:



(5) Again $g(x) = \frac{1}{x^2-4}$



Remember: $x \to 2^+$ means: x approaches 2 from above. That is, x decreases from 100,000, to 50,000, to 10, to 3, to 2.8, to 2.1 to 2.01 to 2.0001 etc.

for (5) Did you consider:
$x \to 2^-$, $x \to -2^+$,
$x \to -2^-$, $x \to +\infty$ $x \to -\infty$?

(6) Graph the following

A) $A(x) = \frac{1}{x^2+4}$

B) $B(x) = \sqrt{x}$

c) $C(x) = -\sqrt{x}$

D) $D(x) = \sqrt{-x}$

E) $E(x) = \sqrt{2-x}$

F) $f(x) = \sqrt{x-2}$

g) $g(x) = \sqrt{4-x^2}$

h) $h(x) = \frac{1}{\sqrt{x}}$

i) $I(x) = \frac{x}{x+1}$

NOTE: $\frac{x}{x+1} = 1 - \frac{1}{x+1}$.
So $x > -1$ $\frac{1}{x+1} > 0$
Thus $I(x) < 1$. What is the EQUATION of the ASYMPOTE? _____.
DON'T FORGET to consider $x < -1$.

J) $J(x) = \frac{x}{x^2-x}$

K) $K(x) = \sqrt{\frac{x}{(x-1)^2}}$

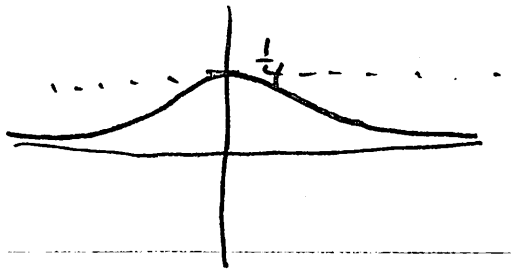
L) $L(x) = \frac{x^2}{x-1}$

(6)

$A(x) = \frac{1}{x^2+4}$

D_A $x \in \mathbb{R}$

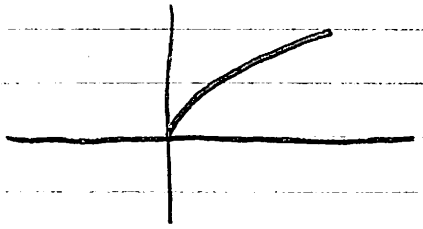
R_A $A(x) \leq \frac{1}{4}$



$B(x) = \sqrt{x}$

D_B $x \geq 0$

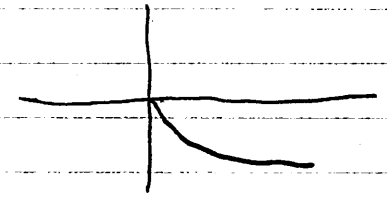
R_B $B(x) \geq 0$



$C(x) = -\sqrt{x}$

D_C $x \geq 0$

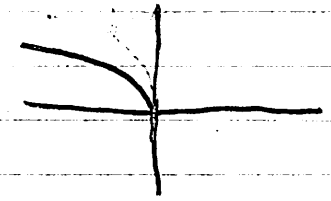
R_C $C(x) \leq 0$



$D(x) = \sqrt{-x}$

D_D $x \leq 0$

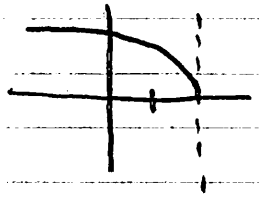
R_D $D(x) \geq 0$



$E(x) = \sqrt{2-x}$

D_E $x \leq 2$

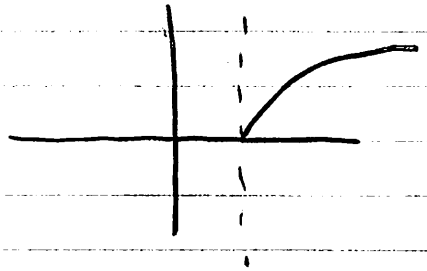
R_E $E(x) \geq 0$



$f(x) = \sqrt{x-2}$

D_F $x \geq 2$

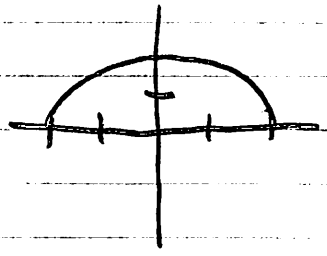
R_f $f(x) \geq 0$



$g(x) = \sqrt{4-x^2}$

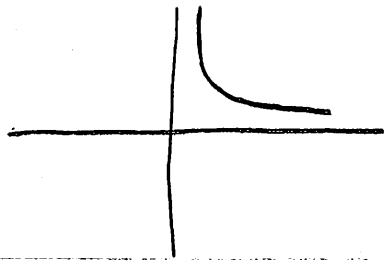
D_g $|x| \leq 2$

R_g $0 \leq g(x) \leq 2$



(more) ↓

h) $h(x) = \frac{1}{\sqrt{x}}$



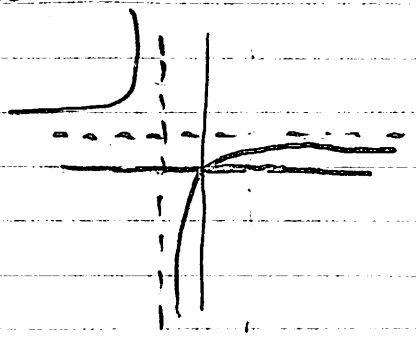
D_h $x > 0$

R_h $h(x) > 0$

as $x \to 0^+$ $h(x) \to +\infty$

as $x \to +\infty$ $h(x) \to 0$

i) $i(x) = \frac{x}{x+1}$



D_I $x \neq -1$

R_I $i(x) \neq 1$

as $x \to +\infty$ $I(x) = 1$

as $x \to -1^+$ $I(x) \to -\infty$

as $x \to -\infty$ $I(x) = 1$

as $x \to -1^-$ $I(x) = +\infty$

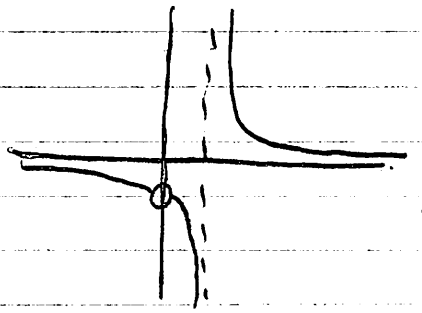
$x = \frac{z}{z+1}$

$xz + x - z = 0$

$z(x-1) = -x$

$z = \frac{-x}{x-1}$

j) $J(x) = \frac{x}{x^2 - x} = \frac{1}{x-1}$



D_J $x \neq 1, 0$

R_J $J(x) \neq 0$

as $x \to +\infty$ $J(x) \to 0$

as $x \to 1^+$ $J(x) \to +\infty$

as $x \to 1^-$ $J(x) \to -\infty$

as $x \to 0^+$ $J(x) \to -1$

as $x \to 0^-$ $J(x) \to -1$

as $x \to -\infty$ $J(x) \to 0^-$

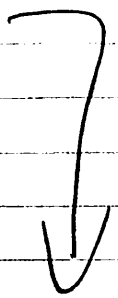
$x = \frac{1}{z-1}$

$xz - x = 1$

$xz = 1 + x$

$z = \frac{1+x}{x}$

k



4) $K(x) = \sqrt{\frac{x}{(x-1)^2}} = \frac{\sqrt{x}}{|x-1|}$ 2 cases

D_K $x \geq 0, x \neq 1$ case I if $0 \leq x < 1$ then $K(x) = \frac{\sqrt{x}}{1-x}$

R_K $K(x) \geq 0$

as $x \to 0^+$ $K(x) \to 0$

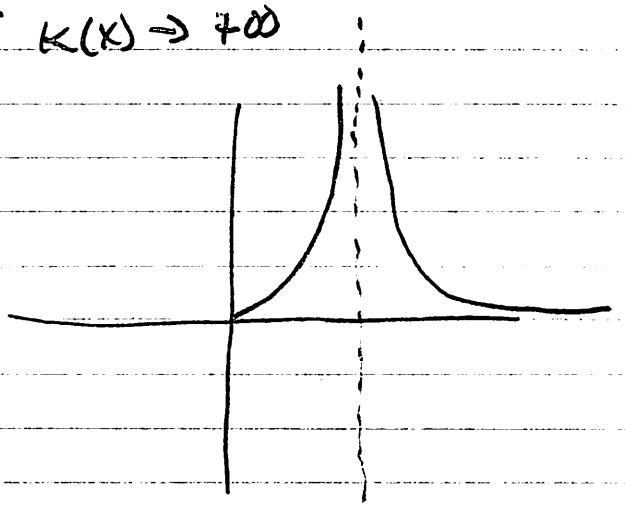
as $x \to 1^-$ $K(x) \to +\infty$

case II if $x > 1$

$K(x) = \frac{\sqrt{x}}{x-1}$

as $x \to 1^+$ $K(x) \to +\infty$

as $x \to +\infty$ $K(x) \to 0$



5 $L(x) = \frac{x^2}{x-1} = x+1+\frac{1}{x-1}$

D_L $x \neq 1$

R_L

$x \to 1^+$ $L(x) \to +\infty$

$x \to 1^-$ $L(x) \to -\infty$

$x \to +\infty$ $L(x) \to x$

$x \to -\infty$ $L(x) \to -x$

$\frac{x}{1-\frac{1}{x}}$

$1 = \frac{x^2}{x-1}$

$x-1 = x^2$

$x^2 - x + 1 = 0$

$(x-2)$

$x(x-4) > 0$ $x = \frac{2}{2-1}$

$x > 0$ $x^2 - x2 + x = 0$

$x < 0$ $x < 4$ $x = \frac{x \pm \sqrt{x^2 - 4x}}{2}$

$x+1$ $x + \frac{x}{x-1}$

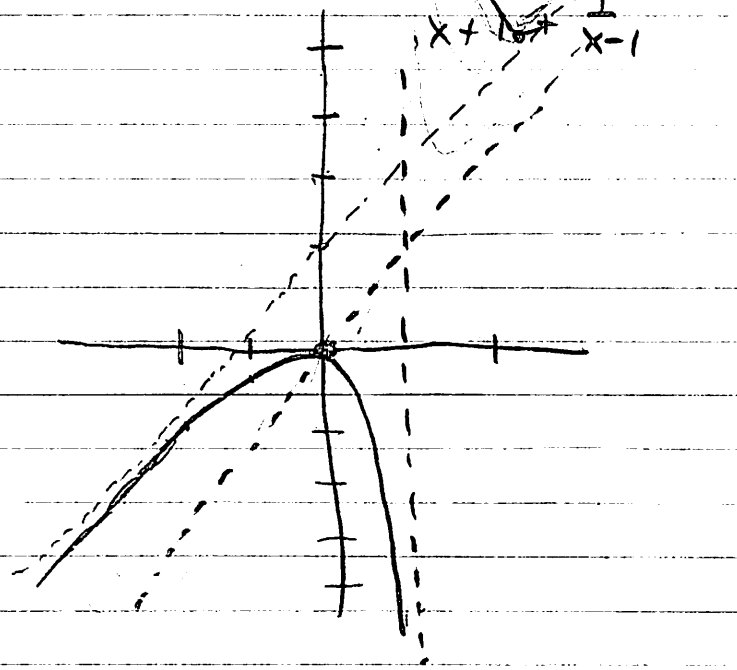
$x-1 \overline{)x^2}$

$x^2 - x$

x

$x - 1$

$x + 1 + \frac{1}{x-1}$



$f(x) = \frac{x+1}{x-2}$

D_f obviously $x \neq 2$

R_f ?

We use the inverse function to assist
Because "Domain Restrictions on the original = Range restrictions on the inverse AND Range Restrictions on the Original = Domain Restrictions on the inverse.

Since we can get Domain restrictions "by inspection" we find the inverse and determine its Domain restriction. Since these are the same as the Range restriction of the original we can easily state R_f.

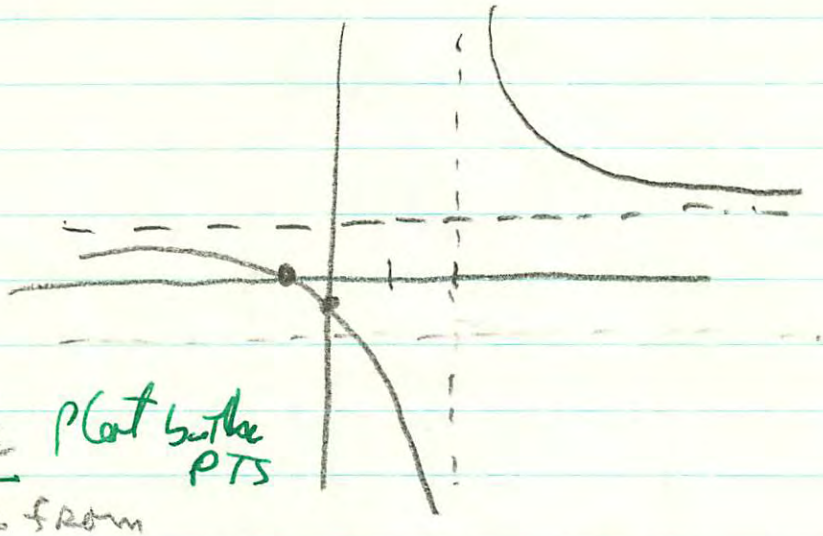
2 Step method of getting inverse.
1. Replace x's with z and f(x) name with x.
2. Solve for z.

$x = \frac{z+1}{z-2}$

$xz - 2x = z + 1$
$xz - z = 2x + 1$
$z = \frac{2x+1}{x-1}$ D_z $x \neq 1$ $\therefore R_f$ $f(x) \neq 1$ More ↓

Significance of Range & Domain Restrictions.

$$f(x) = \frac{x+1}{x-2}$$



D_f $x \neq 2$

R_f $f(x) \neq 1$

$\frac{x+1}{x-2} \neq$ (0) ✓ (−1) ✗

when $x=0$ $f(x) = -\frac{1}{2}$ Plot both pts

as x gets close to 2 from Bigger than 2 f(x) gets very positive

as x goes toward +∞ f gets close to 1 [DRAW upper right portion

as x gets closer to 2 from smaller than 2, f(x) <0. as a matter of fact we have $\frac{+}{-}$ and the bottom is a neg fraction smaller than one so ENTIRE FCTN value is very Negative

as x goes toward −∞ Both top & bottom are Neg so f is + # Near 1.

[DRAW lower left portion

Division does have its usual problem. Given $f(x)$, $g(x)$ then the Quotient of f and g is defined to be $\frac{f(x)}{g(x)}$. Provided $g(x) \neq 0$.

example #1 $f(x) = x+5$ $g(x) = x-1$

$D_f = D_g = \mathbb{R}$

$$\frac{f(x)}{g(x)} = \frac{x+5}{x+1}, \quad x \neq 1$$

ex #2 $f(x) = (x+1)(x+2)$ $g(x) = x+1$

$$\frac{f(x)}{g(x)} = h(x) = \frac{(x+1)(x+2)}{(x+1)} = x+2, \quad x \neq -1$$

let $J(x) = x+2$

Notice $J(x) \neq h(x)$ since the Domains do not agree.

arithmetic operations over fctns.

addition eg. $f(x) = 2x$ $g(x) = 1-x$ =
then $f(x) + g(x) \equiv 2x + 1 - x = x + 1 = h(x)$
multiplication $f(x) \cdot g(x) \equiv (2x)(1-x) = 2x - 2x^2 = h(x)$

the Domain of the new fctn $h(x)$ is defined to be the intersection of the domains of $f(x)$ and $g(x)$ or look at the Domain being all Reals less the union of the restrictions

Scalar multiplication is the multiplication of a fctn $f(x)$ by a constant term c. Let $K(x) = c$ then scalar multiplication becomes or is recognized as simply multiplication of fctns.

Subtraction is defined to be addition of a negative quantity. Given $f(x)$ and $g(x)$. Through scalar multiplication, letting $c = -1$ we have $-g(x)$. Result: $f(x) - g(x)$.

the reason for lumping these all together is now we do not have to redefine the resulting Domains of all our operations.

In Overhead

Quiz 2pts ea

The Domain of the following is the Real # with some exceptions. List those exceptions ~~if any~~ or write None

$f(x) = \frac{x+1}{x+1}$ None

$g(x) = \frac{\sqrt{x+3}}{x^2+x}$ $x \geq -3$, $x \neq -1$, $x \neq 0$

$h(x) = \frac{\sqrt{1-x^2}}{\sqrt{x}}$ $x > 0$, $x \neq 1$

Since $x = -3, -2$ are not included in the $x \geq 0$ category

$j(x) = \frac{x\sqrt{x}}{x^2+5x+6}$ $x \geq 0$

$2x^2+3x+7$

$(\quad)(\quad)$

$\frac{-3 \pm \sqrt{4-56}}{8}$

$K(x) = \frac{x\sqrt[3]{x}}{x^2+5x+6}$ $x \neq -2, -3$

$L(x) = \frac{1}{x-3}$ $x \neq 3$

$m(x) = \frac{2-x}{2x^2+3x+7}$ None

$N(x) = \frac{4}{3x^2+2x-3}$ $x \neq \frac{-1 \pm \sqrt{10}}{3}$

$p(x) = \frac{3x}{3x+1}$ $x \neq -\frac{1}{3}$

$Q(x) = \frac{4x}{7}$ None

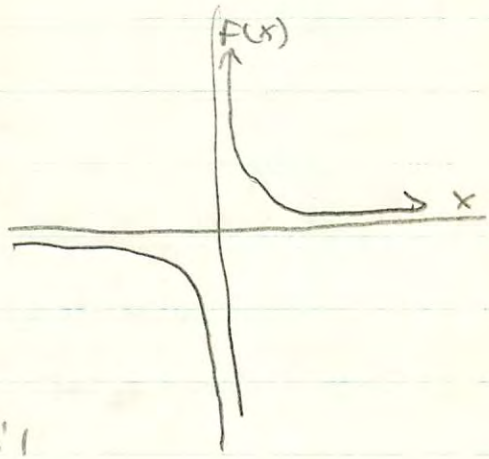
Rons 2pts. are these fctns equal? Explain.

$R(x) = \frac{(x+1)(x+2)}{(x+2)(x+3)}$

$S(x) = \frac{x+1}{x+3}$

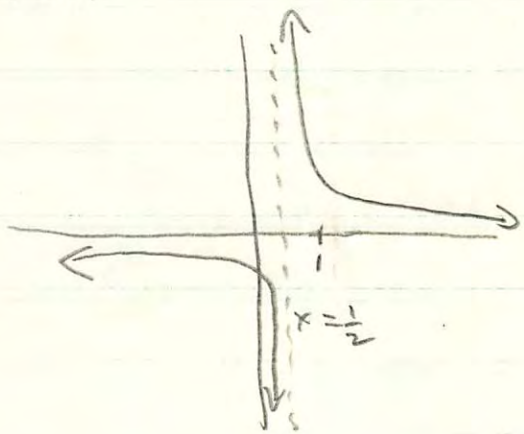
what do these fcns look like?
This is one of uses of the information that we collected about these fcns,
1, 3, 5 are straight lines.

7 $f(x) = \frac{1}{x}$ D_f $x \in \mathbb{R}$ $x \neq 0$
R_f $f(x) \in \mathbb{R}$ $f(x) \neq 0$



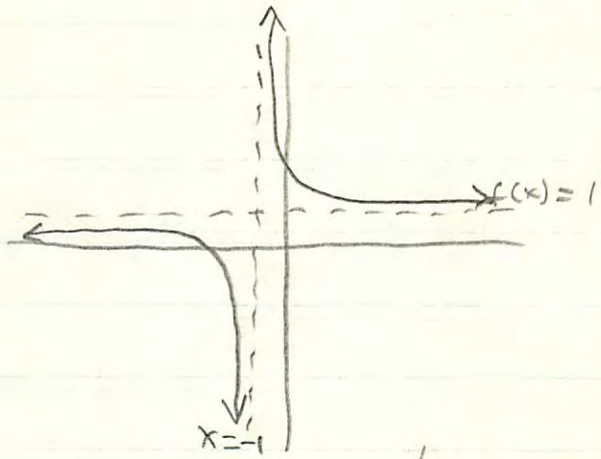
clearly x axis is asymptote, this is called the horizontal asymptote
consider $x \to 0^+$ $f(x) \to \infty$
$x \to +\infty$ $f(x) \to 0^+$ f is 1:1
$x \to 0^-$ $f(x) \to -\infty$
$x \to -\infty$ $f(x) \to 0^-$

(9) $f(x) = \frac{1}{2x-1}$ D_f $x \neq \frac{1}{2}$
R_f $f(x) \neq 0$



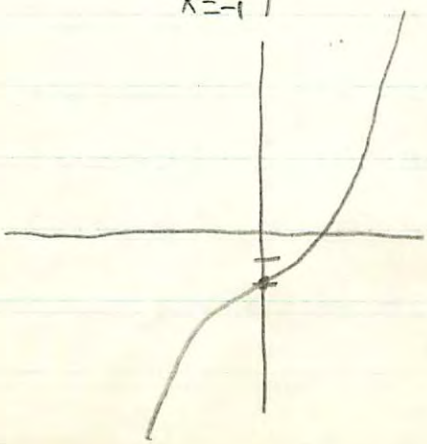
as $x \to +\infty$ $f(x) \to 0$
$x \to \frac{1}{2}^+$ $f(x) \to +\infty$
$x \to -\infty$ $f(x) \to 0^-$
$x \to \frac{1}{2}^-$

(11) $f(x) = \frac{x}{x+1}$ D_f $x \neq -1$
R_f $f(x) \neq +1$



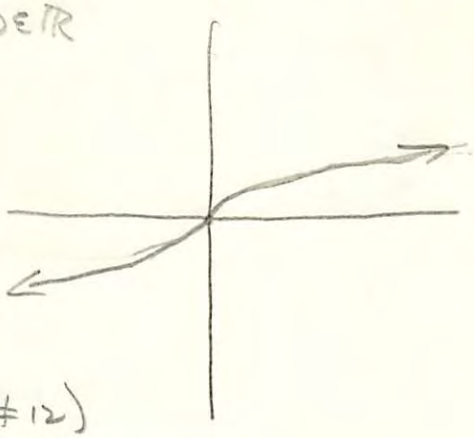
as $x \to +\infty$ $f(x) \to 1$
as $x \to -1$ $f(x) \to$

(13) $f(x) = x^3 - 2$ D_f $x \in \mathbb{R}$
R_f $f(x) \in \mathbb{R}$



$x = 0$ $f(x) = -2$
$x \to +\infty$ $f(x) \Rightarrow +\infty$
$x \to -\infty$ $f(x) \Rightarrow -\infty$

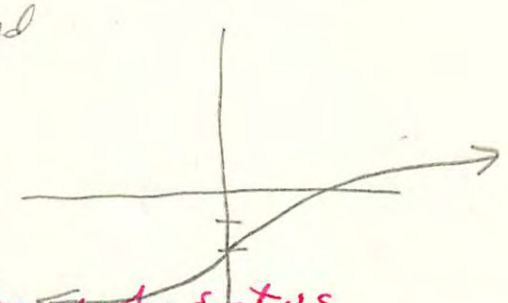
(15) $f(x) = x^{\frac{1}{3}}$ D_f $x \in \mathbb{R}$ R_f $f(x) \in \mathbb{R}$



$x = 0$ $f(x) = 0$
$x \to 0^+$ $f(x) \to 0^+$
$x \to 0^-$ $f(x) \to 0^-$
$x \to +\infty$ $f(x) \to +\infty$
$x \to -\infty$ $f(x) \to -\infty$

Notice this is simply the inverse of $g(x) = x^3$ (Pblm #12)
Notice curve is not asymptotic

(17) $f(x) = x^{\frac{1}{3}} - 2$ Just #15 lowered 2 units



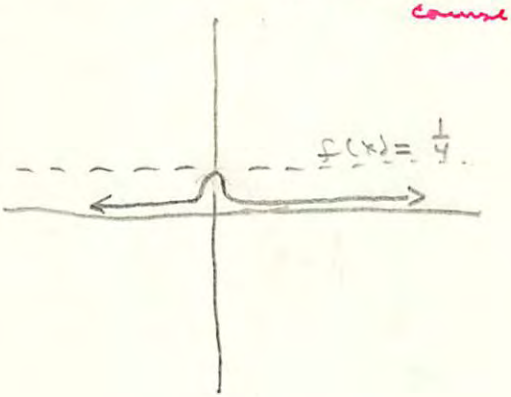
END of 1st discussion on the graphs of the fctns.
In addition to those pblms already assigned students should Sketch the graphs of f and g as defined $f(x) = \frac{1}{x^2+4}$, $g(x) = \frac{1}{x^2-4}$
Lets consider some other curves. Domain + Range of curve.

$f(x) = \frac{1}{x^2+4}$ D_f $x \in \mathbb{R}$
R_f $x < \frac{1}{4}, x \neq 0$

$x = \frac{1}{z^2+4}$

$xz^2 + 4x = 1$
$xz^2 = 1 - 4x$
$z^2 = \frac{1-4x}{x}$
$z = \pm\sqrt{\frac{1-4x}{x}} \to x \neq 0$

$1 - 4x > 0$
$1 > 4x$
$\frac{1}{4} > x$

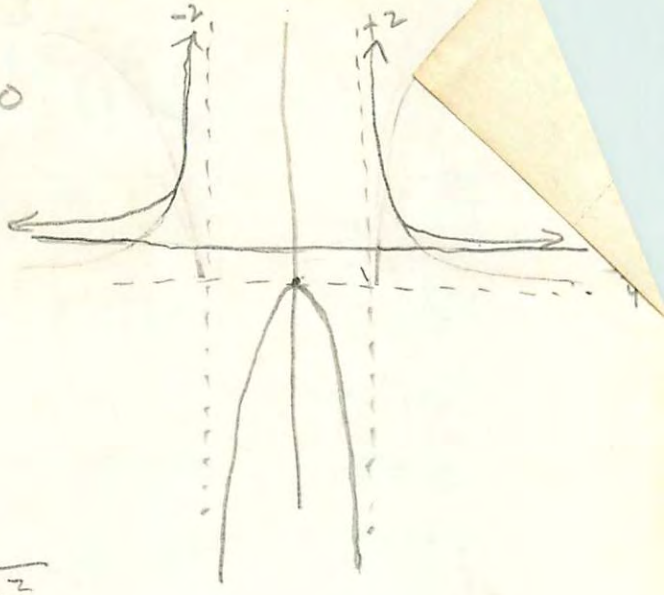


as $x \to 0^+$ $f(x) \Rightarrow \frac{1}{4}$
$x \to 0^-$ $f(x) \to \frac{1}{4}$
$x \to +\infty$ $f(x) \to 0$
$x \to -\infty$ $f(x) \to 0$

compare this fctn with $g(x)$ which follows.

$g(x) = \frac{1}{x^2-4}$ $\quad D_g \quad x \neq \pm 2$

$x = \frac{1}{z^2-4}$ $\quad R_g \quad g(x) > -\frac{1}{4}, g(x) \neq 0$



$xz^2 - 4x = 1$

$xz^2 = 1+4x$ $\quad 1+4x > 0$

$z^2 = \frac{1+4x}{x}$ $\quad x > -\frac{1}{4}$

$z = \pm\sqrt{\frac{1+4x}{x}}$ $\quad x \neq 0$

$\sqrt{\frac{1}{x}+4}$

as $x \to +\infty$ $\quad g(x) \to +\infty$

$x \to -\infty$ $\quad g(x) \to +\infty$

$x \to 2^+$

$\frac{\sqrt{x+4x^2}}{x}$

$\frac{\sqrt{x(1+4x)}}{x}$

$x > 0$ & $1+4x > 0$

$x > 0$

$x < 0$ & $1+4x < 0$

$x < -\frac{1}{4}$

Need to rationalize denominator. In order to (accurately) determine restrictions on Range of g!

Before pg 45-46 (1)

$f(x) = \dfrac{3x+2}{4x-1}$ $G(x) = 2|x-3|+1$ $H(x) = \sqrt{(x+1)(x-2)}$

Domain "by inspection" or whatever...

D_f $x \neq \frac{1}{4}$ D_G $x \in \mathbb{R}$

+ 0 − 0 +
−1 2

D_H $x \leq -1$ or $x \geq 2$

What about Range?

for G, Range must be R_G $G(x) \geq 1$ (because $2|x-3|$ the least candie is zero)

for H, Range must be R_H $H(x) \geq 0$ since a sq root is ≥ 0.

for F we need the inverse function to help us.

The INverse — 2 Step Process

$f(x) = \dfrac{3x+2}{4x-1}$

1st step $x = \dfrac{3z+2}{4z-1}$

2nd step solve for z

$4xz - x = 3z + 2$

$(4x-3)z = x+2$

$z = \dfrac{x+2}{4x-3}$

$\rightarrow D_{f^{-1}}$ $x \neq \frac{3}{4}$ $\therefore R_f$ $f(x) \neq \frac{3}{4}$

Another use of inverse fcn is to prove a fcn is 1:1

$f(f^{-1}(x)) = x = f^{-1}(f(x))$

usually, one side is enough.

↓

$f(x) = \frac{3x+2}{4x-1}$ $\quad f^{-1}(x) = \frac{x+2}{4x-3}$

$$f(f^{-1}(x)) = \frac{3\left(\frac{x+2}{4x-3}\right)+2}{4\left(\frac{x+2}{4x-3}\right)-1}$$

$$\frac{\frac{3x+6}{4x-3}+2}{\frac{4x+8}{4x-3}-1}$$

$$\frac{\frac{3x+6+8x-6}{4x-3}}{\frac{4x+8-4x+3}{4x-3}}$$

$$\frac{11x}{11}$$

$$x$$